Chapter 5

NEWTON'S LAWS

To this point, we've dealt with definitions and kinematics but haven't really done anything that has had a solid technique associated with it. Newton's Laws will be the first. Although we will play with all three of Newton's Laws, the second law is the work horse.   In a nutshell, it maintains that if you can identify the forces acting on a body, you can deduce the body's acceleration. It sounds simple, but it will provide you with a very powerful approach to analyze situations.

A.) Newton's Three Laws:

1.) <u>Newton's First Law:</u> In an inertial frame of reference, bodies in motion tend to stay in motion in a straight line, and bodies at rest tend to stay at rest, unless impinged upon by a net external force.

a.) Example: When an object in space is given a quick push and then left alone, it will move with a constant velocity in the direction of the push until an outside (external) force accelerates it into another kind of motion (i.e., makes it go faster, slower, change directions, or some combination thereof).

Until a net external force is applied, the constant-velocity, straight-line motion of a body will continue unchanged.

2.) <u>Newton's Second Law:</u> The *acceleration **a*** of a body (as a vector) is proportional to the *net force **F*** (also a vector) acting on the body.

a.) Mathematically, this can be stated as

$$\mathbf{F} = m\mathbf{a},$$

where the proportionality constant *m* is the *mass of the object being accelerated* (see section on "What is MASS" at the end of this chapter).

b.) As a force in the *x direction* will not make a body accelerate in the *y direction*, we can break both the *net force* and *acceleration* into their component parts and write three direction-related force equations:

$$F_{net,x} = m\,(a_x), \qquad\qquad F_{net,y} = m\,(a_y), \qquad\qquad F_{net,z} = m\,(a_z).$$

Note 1: These look like scalar equations because we haven't included unit vectors to designate direction, but each, nevertheless, represents a vector quantity. As such, we need to be careful of *direction* as denoted by *positive* and *negative* signs.

Note 2: Newton's Second Law is the work-horse of the three laws. It is a very powerful tool for analyzing situations in which forces and accelerations are involved. There is a technique involved in its use, outlined in *Part B*. LEARN THE APPROACH. Understand it. You should become so familiar with it that you can use it to evaluate problems *you have never seen before* using nothing more than the approach and your head.

3.) <u>Newton's Third Law:</u> For every force in the universe, there exists, somewhere, an *equal* and *opposite* reaction force.

Note: This idea of an *action/reaction* pair is somewhat misleading as it suggests that one force follows the other. As the examples will show, that is not the case. We will use the terminology because it is standard, not because it is an intelligent way of characterizing the interaction.

a.) Example 1: Imagine hitting a door with your hand. Doing so applies a force to the door which, if great enough, will break the door. In turn, the door applies a force to you which, if great enough, will break your hand. The size and direction of the force you apply must, according to N.T.L., be equal and opposite the force it applies to you.

i.) In the case of the door, the *action* is characterized by the statement, "You apply a force to the door" while the *reaction* is characterized by the statement, "The door applies a force to you." Notice that the wording is almost identical with the exception of reversed noun and/or pronoun order. This is always the case with the characterization of action/reaction pairs.

ii.) Don't be misled by the apparent personification of "the door." It obviously hasn't made a conscious decision to hit you back, even though the characterization implies that such be the case. Newton's Third Law is a commentary on the *structural reality* of the universe. The only way forces can be generated are through action/reaction pairs. By implication, if you try to apply 180 pounds of force to a table that can only provide 150 pounds of reaction force back on you, you will never succeed. The table will give way as soon as you exceed the 150 pounds that it can apply to you.

b.) Example 2: If the *earth* applies a gravitational force on *you*, what is the reaction force that must, according to N.T.L., exist?

  i.) Answer: The reaction force is the gravitational force that *you* exert on the *earth*. Again, the same rearranged language is used to state the *reaction force* as was used to state the *action force*.

  c.) Example 3: A truck runs head-on into a train. The train is ten times more massive than the truck. How do the forces experienced by each compare?

  i.) The answer is, THEY ARE THE SAME! The train applies a force to the truck and, as a consequence, experiences an acceleration. The truck applies a force to the train and, as a consequence, experiences an acceleration. The two accelerations are different but, according to N.T.L., the two forces *must be* equal. Figures 5.1a and 5.1b depict this situation along with the fact that as *ma* equals the force on each body, that *product* must be the same for each body.
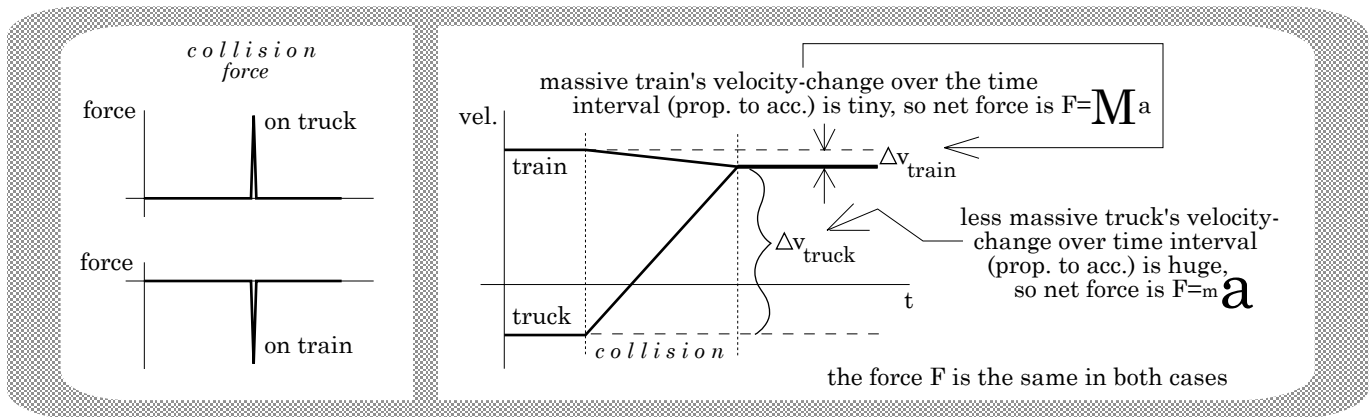

FIGURE 5.1a   FIGURE 5.1b

B.) Newton's Second Law and Types of Forces:

1.) *Newton's Second Law* is intimately related to force acting on a body. A brief preliminary discussion of the five kinds of forces you will work with follows in *Parts 2* through *6*:

2.) <u>Gravitational force:</u>

  a.) Gravity is a force of attraction between any two massive objects.

  b.) When the earth is one of the two bodies involved:

i.) The gravitational force felt by the second object (you, for instance) while positioned on the earth's surface will always be directed toward the earth's center.

ii.) The gravitational force on the body will have a magnitude equal to the product *mg*, where *m* is the mass of the object in question and *g* is the gravitational acceleration near the earth (9.8 m/s^2 in the MKS system of units; 980 cm/s^2 in the CGS system of units; 32.2 ft/s^2 in our sytem of units--this is called *the English* system).

Note: There are two metric systems of units (the MKS and the CGS systems) and one non-metric system (our own--the "English" system):

The MKS system uses *meters* for length, *kilograms* for mass, and *seconds* for time (hence, MKS). Force is in *newtons* and energy is in *joules*.

The CGS system uses *centimeters* for length, *grams* for mass, and *seconds* for time (hence, CGS). Force is in *dynes* and energy is in *ergs*.

The English system will never be used in this book (a base-10 version is used in engineering applications, but physicists generally stay away from it). For completeness, though, it uses *feet* for length, *slugs* for mass, and *seconds* for time. Force is in *pounds* and energy is in *foot-pounds*.

NEVER MIX UNIT-SYSTEMS. If you find yourself with mass information in grams (the CGS system) and you want to know the body's weight, do not multiply the mass by 9.8 m/s^2--the MKS value for the acceleration of gravity. Either convert the mass to kilograms (divide by 1000) or use the CGS value for *g* (i.e., 980 cm/s^2).

iii.) The gravitational force will be equal to the body's weight (that's right, folks, when you step on a scale and it measures your weight, you are really measuring *mg*--the force of attraction between you and the earth). As weight is a *force*, its units in the MKS system are *kilogram·meter/second^2*, or *newtons*.

c.) Example: Assuming we neglect air friction, a free falling object has only one force acting on it--gravity (see Figure 5.1c).

3.) <u>Normal force:</u>

a.) A normal force is a force of support provided to a body by a surface in which the body is in contact;

b.) Normally characterized by *N*, a *normal force* is always directed *perpendicular to* and *away from* the surface providing the support;
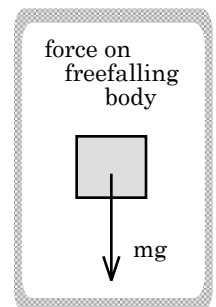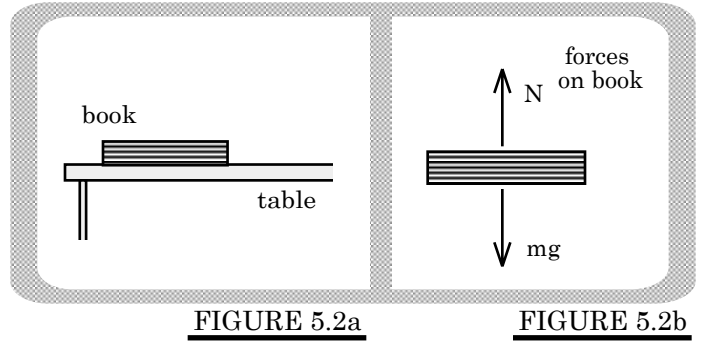

force on freefalling body

mg

FIGURE 5.1c

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c.) Example 1: In Figure 5.2a, a book is supported by a table. Figure 5.2b depicts all the forces acting on the book (this is called a *free body diagram*, or *f.b.d.*). Notice that the *normal force* acting on the book is *perpendicular to* and *away from* the table's top (the *gravitational force* is also shown).

FIGURE 5.2a   FIGURE 5.2b

d.) Example 2: In Figure 5.3a, a book rests on an incline plane. Figure 5.3b shows the f.b.d. for the forces acting on the book. As was the case in Example 1, the *normal force* acting on the book is *perpendicular to* and directed *away from* the in-cline's top, even though the top is not in the horizontal!

FIGURE 5.3a   FIGURE 5.3b
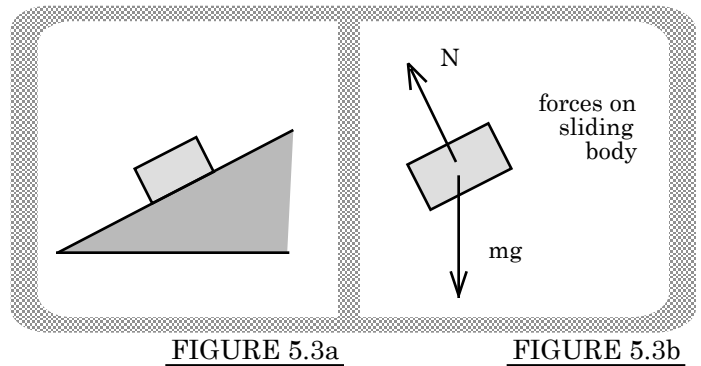
Note: The *normal force* acts on the under-side of the book, as shown in Figure 5.4. Though technically dubious, *normal forces* are usually presented as in Figure 5.3b. This is done for con-venience (you will see why shortly). Either posi-tioning is acceptable.

FIGURE 5.4
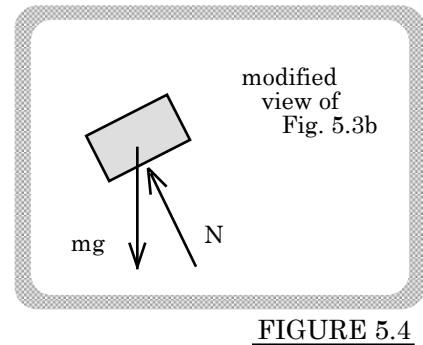
4.) Tension force:

a.) Tension force is applied to a body by a rope, string, or cable.

b.) Normally characterized by T, tension forces are always applied *along the line of the cable* and *away from* the body in question.

c.) Example: A string is attached to a hanging mass, threaded over a pulley, and attached to a second mass sitting on a table (see Figure 5.5a). The forces on the *hanging mass* are shown in Figure 5.5b; the forces on the *tabled mass* are shown in Figure 5.5c. In both cases, the *tension force* is directed *away from the body* and *along the line of* the string.
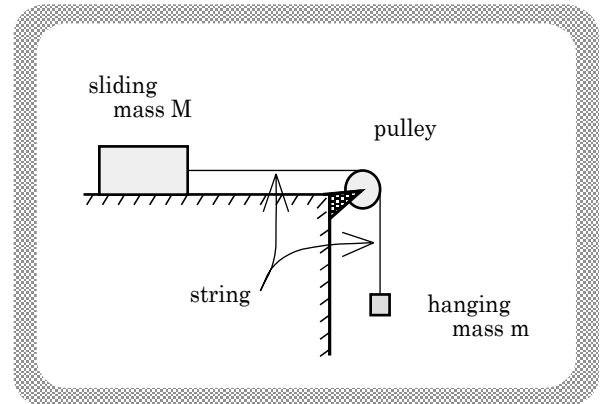

FIGURE 5.5a

Note: If the pulley is one of those mythical "frictionless, massless" jobs assumed-into-existence by physics departments across the country, the *massless* nature of the pulley will allow the *magnitude of the tension force* on either side of the pulley to be the same. In other words, an ideal pulley changes the DIRECTION of the tension force (in this case, from the horizontal to the vertical) but does not change the force's magnitude.
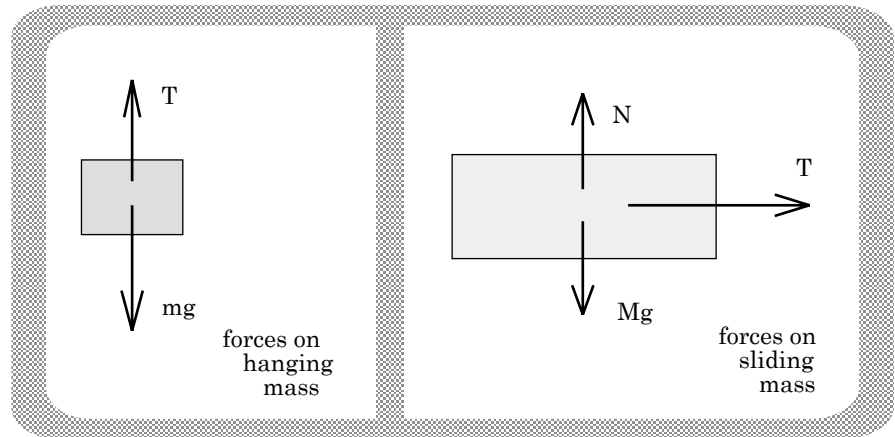

FIGURE 5.5b

FIGURE 5.5c

5.) Frictional force:

a.) Friction is produced by the atomic interaction between two bodies as they either slide over one another (this is called *kinetic friction*) or sit motionless in contact with one another (this is called *static friction*).

Note: When two bodies come very close (i.e., rest against one another), there is a weak atomic bonding that occurs between the electrons of the one structure and the protons of the other (and vice versa). It is almost as though the atoms of the two bodies have melded to some degree. When the bodies try to move over one another, this bonding has to be sheared. That shearing is what produces the retarding effects we call friction.

b.) Kinetic friction: Sometimes called *sliding friction*, kinetic friction occurs when one body slides over or against a second body. The direction of *kinetic friction* is always *opposite* the direction of motion.

From experimentation, it has been observed that the amount of *kinetic friction* f_k a body experiences is proportional to the size of the *normal force N* exerted on the body by the structure it slides against. Mathematically, these two parameters (f_k and N) are related as:

$$f_k = \mu_k N,$$

where μ_k is a proportionality constant called the *coefficient of* <u>*kinetic*</u> *friction*.

As μ_k is normally given in a problem, all that is required to calculate the magnitude of the *kinetic frictional force* on a body is the *normal force* exerted by the supporting structure.

Note: The statement *kinetic friction is a function of normal force only-- surface area has nothing to do with it* is true ONLY as long as you are dealing with two rigid bodies that are sliding relative to one another.

The tires of a dragster do NOT fit this bill. Dragster drivers spin their tires before a race to make them sticky. As such, the friction analogy for race tires is more like the dragging of a piece of Scotch tape across a desk than the sliding of rigid bodies--the bigger the surface area, the greater the stickiness and traction. My gratitude to Jim Malone (Mercersburg Academy) for contacting Mike Trinko of Goodyear's Race Group for this clarification.

c.) Example of *kinetic friction*: A block slides down a frictional surface (see Figure 5.6a). The *coefficient of kinetic friction* μ_k is known. Figure 5.6b shows all the external forces acting on the block.
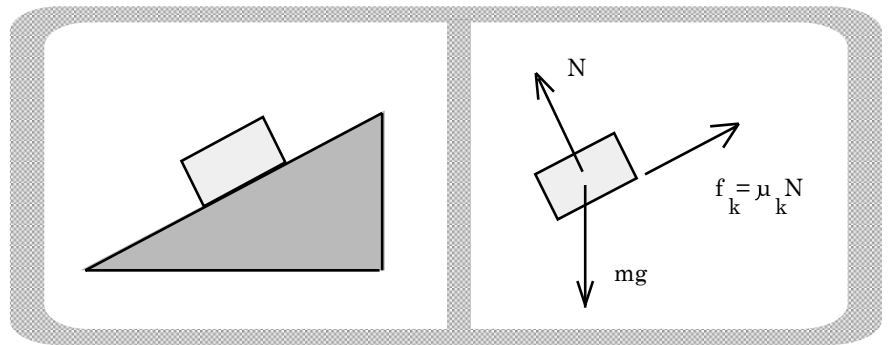

FIGURE 5.6a

FIGURE 5.6b

Note: Notice that the normal force N sometimes equals the weight mg of the sliding body (an example of this can be seen in Figure 5.2b), BUT NOT ALWAYS. For the incline plane depicted above, N and mg are not even in the same direction.

d.) <u>Static friction:</u> If a book, for instance, sits on a table and a tiny external force is applied to the book *parallel to the table top*, the force may or may not move the book. There will be a force-of-opposition in this case provided by *static friction*. Its cause is the same as the partial atomic bonding that produces *kinetic friction* with one exception; static frictional bonding is stronger (with the two surfaces stationary relative to one another, the atoms meld more deeply requiring a greater shearing force to separate them).

If successively harder and harder forces are applied, the static frictional force will counter each push with a force of equal and opposite magnitude until the *applied force* is great enough to shear the partial bonding between the surfaces. At that point, the book will break loose and begin to slide.

It has been experimentally observed that this <u>*maximum*</u> static frictional force $f_{s,max}$ is proportional to the *normal force N* applied to the book by the table. Mathematically, these two parameters ($f_{s,max}$ and N) can be related as:

$$f_{s,max} = \mu_s N,$$

where μ_s is called the *coefficient of <u>static</u> friction*.

Note: Most physics books do not write the *maximum static frictional force* as $f_{s,max}$. Instead, they simply write f_s. For simplicity, we will do the same. Be clear, though. If you know f_s, you know only <u>*one*</u> *of the infinitely many* possible static frictional forces that could be exerted between the two bodies. Which force do you know? The <u>*maximum*</u> *static frictional force*.

On the other hand, if you know f_k you know the single, CONSTANT *kinetic frictional force* that exists between two bodies sliding relative to one another. No matter what their velocity (assuming heating doesn't change the characteristics of the two surfaces and alter the *coefficient of kinetic friction*), the *frictional force* f_k will always be the same.

The two quantities--f_s and f_k-- look similar as far as notation goes, but they tell us two different things!

6.) <u>Push-me, pull-you force:</u>

a.) Any force that does not fall into one of the above categories falls into this one. Ex: A shove from a friend. The magnitude of such a force is usually characterized by an F, possibly with a subscript.

C.) Newton's Second Law--APPROACHES:

1.) There are two ways to deal with Newton's Second Law problems.

    a.) The first is the formal, technically kosher way to proceed. It has specific steps and works on even the most convoluted force/acceleration problems. These include situations in which forces do not act in the same direction as the motion (e.g., centripetal situations), situations in which several objects within the system experience non-equal accelerations, and, with slight modification, situations in which some of the forces involved are velocity dependent.

    b.) The second approach is a simplified version of the first. When applicable, it makes the evaluation of N.S.L. problems considerably quicker and easier (this can be very useful when dealing with multiple choice test questions). As such, your initial temptation may be to focus on that approach to the exclusion of the first. That would be a serious mistake. Getting the answer isn't important, here. Understanding the approaches is. To that end, both approaches will be presented in the evaluation of the example problems in the next section.

    Note: The *formal approach* is the only approach presented in the Solutions section at the back of the book. If you have correctly used the simplified version on a given problem, you can still use the book-end solutions to check your final answer.

    2.) The simplified approach: When dealing with the simplified approach, the question you want to ask is, "What are the forces that motivate or retard acceleration in the system?" Determine those forces, equate them to *ma*, where *m* is the total mass of the system, and you're done.

    3.) The formal approach: The following is a step-by-step outline for the use of the formal approach associated with Newton's Second Law as a problem-solving tool. Read through quickly, then continue on to see how the technique is used in the Example Problems section. Once you have read through that section, use this outline as a reference whenever you tackle complex Newton's Second Law problems.

    a.) Look at the problem, blanch, think *I couldn't possibly do this*, then begin *the approach* starting with *Step b*.

    b.) Draw a sketch of the entire system if one is not provided.

    c.) Take <u>one body</u> in the system and draw a *free body diagram* (f.b.d.) for it.

i.) A *free body diagram* is a sketch of the body-in-question (it is normally depicted as a box) showing *all the forces acting on the body*. These forces are depicted as arrows. They don't have to be drawn to scale but do have to be directionally accurate (i.e., if the force is in the vertical, don't draw it halfway between the vertical and the horizontal).

Note: F.B.D.'s show only the forces that act *on the body-in-question*. They do *not* show forces that the body applies to other bodies.

ii.) When drawing an f.b.d., be sure the *orientation* of the box representing the body is the same as the actual orientation of the body in the problem. That is, if the object is a crate on an incline, the f.b.d. should depict the object as a box *on a slant*, not as a box in the horizontal--see Figure 5.6.

d.) Choose *x* and *y axes* and place them on your f.b.d. One axis <u>must</u> be in *the direction of the acceleration you are trying to determine*.

Note: This means your axes will not always be in the vertical and horizontal. Think about the direction of acceleration of the crate sketched in Figure 5.6.

e.) If there are forces on the f.b.d. that are not along the *x* and *y* directions, replace them with their *x* and *y components*.

f.) Using N.S.L., sum the forces in the *x* direction and set them equal to the product ma_x. If an additional relationship (equation) is needed, sum the forces in the *y* direction and set them equal to the product ma_y (assuming you are interested in the variables involved).

g.) Repeat the above process for all the bodies in the system *or until you have enough equations to solve for the unknowns you are interested in* in the system.

WARNING: ASSUME NOTHING that can be derived using an f.b.d. and N.S.L.

D.) Example Problems:

The easiest way to become comfortable with both the simplified and formal approaches outlined above is to try a problem. There are two in this section: one that is relatively simple and a second that is more complex. Both have within them potential pitfalls. Look for the sticking points. Understanding

these difficulties will make it easier for you to do other N.S.L.-type problems later on your own (this is especially true of the formal approach).

Note: While you are reading, remember that your goal is to generate equations that relate the *acceleration* of one or more of the masses within the dynamic (i.e., moving) system to the forces acting on that mass.

1.) Example 1--The Run-Away Car using the Formal Approach: Somebody gives your car a shove, then exits. The car rolls freely on a horizontal road toward a cliff. You try to stop the car by applying a known force *F* at a known angle *θ* (see Figure 5.7). If the car's mass is *m*, derive a general algebraic expression for the *acceleration* of the car and, for the fun of it, the *normal force* applied to the car by the ground. We'll use *the formal approach* first.
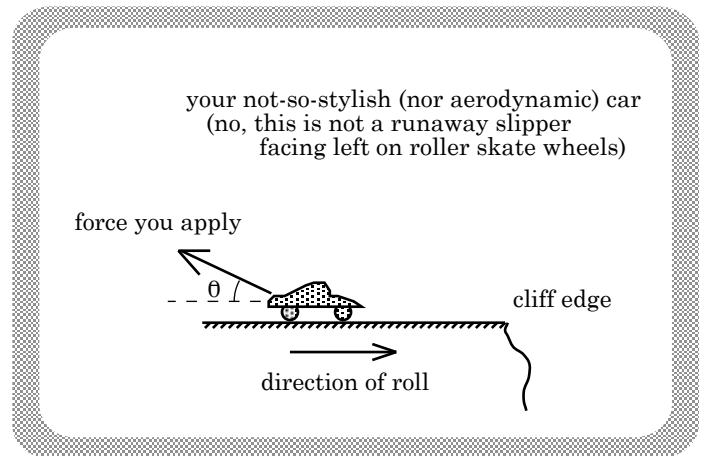
your not-so-stylish (nor aerodynamic) car
(no, this is not a runaway slipper
facing left on roller skate wheels)

force you apply

θ

cliff edge

direction of roll

FIGURE 5.7

--Solution and Approach *By-the-Numbers*:

--Step a: The sketch is seen in Fig. 5.7.

--Step b: A *free body diagram* (f.b.d.) of the forces acting on the car is shown in Figure 5.8.
Notice that the *direction of the car's motion* (i.e., the direction of its velocity vector) is not the same as the *direction of its acceleration*. That means, simply, that the car is slowing down. Notice also that the *velocity vector* is not found on the f.b.d. Only FORCES go on free body diagrams.

F_{you}

N

f.b.d. on car

mg

FIGURE 5.8

--Step c: Figure 5.9 shows the placement of appropriate coordinate axes. One is in the horizontal along the car's *line of acceleration*, and the other is *perpendicular* to the first.
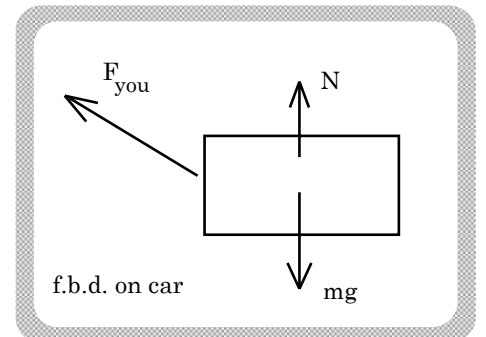
y

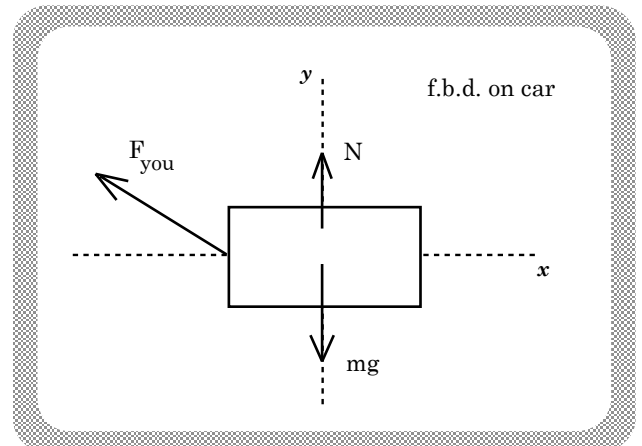f.b.d. on car

F_{you}

N

x

mg

FIGURE 5.9

Note: Although it is customary to make horizontal axes *"+" to the right* and vertical axes *"+" in the upward direction*, you can define any direction as positive.

--Step d: There is one force that is off-axis--the pulling force F. Figure 5.10 shows a revised f.b.d. with F replaced by its components along the two axes.

Note: In Figure 5.10, the vertical component of $\boldsymbol{F}$ (i.e., $F \sin \theta$) is positioned out away from the body, but it actually acts at the same point as does $\boldsymbol{F}$. It has been drawn as part of a right triangle to make its determination easier. Having made that determination, you could re-position the vector as shown in Figure 5.11. There is nothing wrong with doing this, though it does seem a waste of time if its original positioning doesn't confuse you.
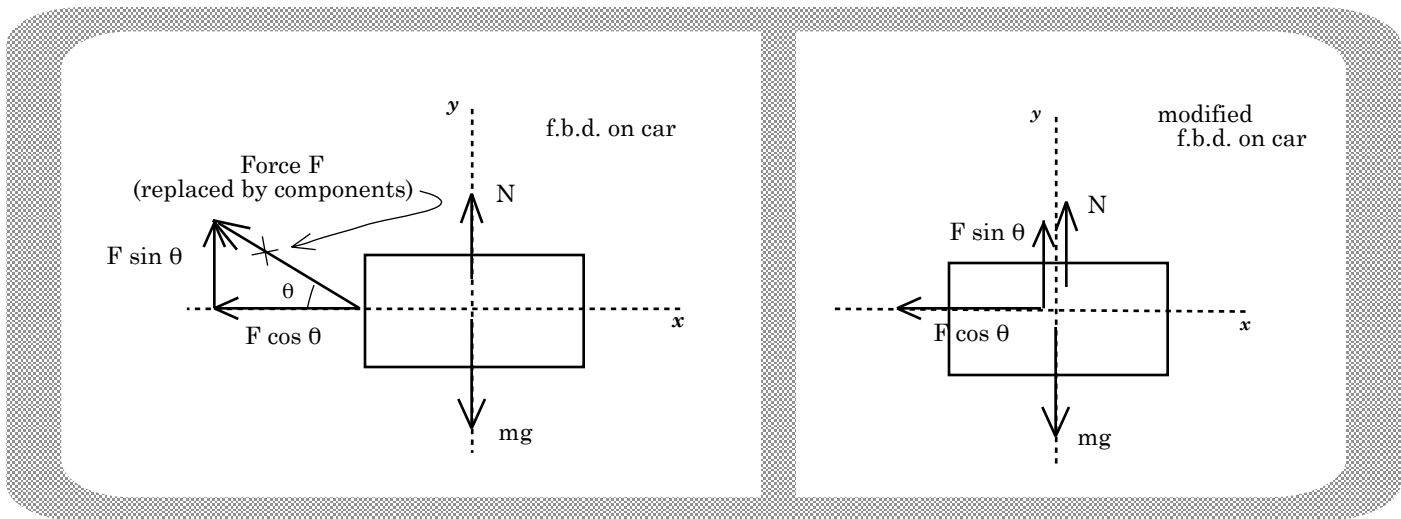

FIGURE 5.10   FIGURE 5.11

--Step e: The *sum of the forces* in the x and y directions is shown below. Notice that each has been preceded by a blurb that denotes what is about to happen. That is, the "$\underline{\sum F_x}$" notation alerts the reader that you are beginning a *sum-of-the-forces-equals-"ma"* process in a given direction. *You will be expected to use this same notation* as a preamble whenever you use Newton's Second Law.

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$$\underline{\sum F_x}:$$

$$-F(\cos\,\theta) = -m(a_x)$$

$$\Rightarrow \quad a_x = F(\cos\,\theta)\,/\,m.$$

Note 1: Notice the *negative sign* in front of the *component of F along the x axis* term? If that force component had been in the opposite direction, its magnitude would have remained *F (cos θ)*, but the sign in front of the component would have been *positive*. In other words, the expression *F (cos θ)* stands for the *magnitude* of the force. When writing out the N.S.L. expression, you have to account for the *direction* of a force by <u>manually</u> inserting the appropriate *positive* or *negative* sign in front of its magnitude value.

Note 2: Just as was the case with the force component discussed in Note 1, the acceleration term a_x can be treated in one of two ways. We can either define it to be a vector, leaving its sign embedded within the symbol, or we can define it to be a *magnitude*, unembed its sign, and manually put that sign in front of *ma*. For reasons that will become obvious later, the latter is preferable.

Note 3: This *unembedding the sign* isn't as scary as it looks. If the assumed direction for the acceleration is incorrect, we will simply end up with an acceleration value that has a negative sign in front of its magnitude when numerically calculated. As magnitudes cannot be negative, the negative sign simply means we have assumed the *wrong direction* for the acceleration.

--back to Step e:

In the *y direction*:

$$\underline{\sum F_y}:$$

$$+F(\sin\,\theta) + N - mg = +m(a_y) \qquad (= 0 \text{ as } a_y = 0).$$

As shown, the acceleration a_y is zero (the car is not hopping up and down off the road). As such, this expression can be re-written as:

$$N = -F(\sin\,\theta) + mg.$$

Note: The normal force exerted by the ground on the car is *not equal to the weight of the car*. This shouldn't be surprising. The car is being pulled

upward by the vertical component of **F**, requiring less *normal force* from the ground to keep the car from falling through the road.

　　–Step f: We have enough equations to solve for the parameters in which we are interested.

　　Using the derived equations with a car mass of 1000 kg, we find that if the angle is 30^0 and the pulling force is 800 nts (you *are* a gooorilla):

$$a_x = F (\cos \theta) / m$$
$$= (800 \text{ nt}) (\cos 30^0) / (1000 \text{ kg})$$
$$= .69 \text{ m/s}^2$$

and

$$N = \text{-}F (\sin \theta) + mg$$
$$= \text{-}(800 \text{ nt}) (\sin 30^0) + (1000 \text{ kg}) (9.8 \text{ m/s}^2)$$
$$= 9400 \text{ nts.}$$

Note: One *newton* is equal to about a quarter of a pound.

　　2.) <u>Example 1--The Run-Away Car using the Simplified Approach:</u> Somebody gives your car a shove, then exits. The car rolls freely on a horizontal road toward a cliff. You try to stop the car by applying a known force *F* at a known angle θ (see Figure 5.7). If the car's mass is *m*, derive a general algebraic expression for the *acceleration* of the car.

　　a.) The question to ask is, "What are the forces that motivate the car to accelerate?" In this case, the only force acting along the line of acceleration is *F (cos θ)*. Putting that equal to ma_x yields:

$$F \cos \theta = ma_x$$
$$\Rightarrow a_x = (F \cos \theta)/m.$$

　　b.) Although this undoubtedly seems wickedly easy in comparison to the more plodding *formal approach* presented in the previous section, remember what we are doing. We are examining relatively simple problems with an eye to developing techniques that will serve you well when you are asked to take apart more complex scenarios. Be patient. As hard as it may be to believe, the *formal approach* is definitely the way to go in many situations.

3.) Example 2--Incline and Pulley using the Formal Approach: A string attached to a known hanging mass m_1 is threaded over a *frictionless, massless pulley* and attached at its other end to a second known mass m_2 that is supported by a *frictional* incline plane. The mass m_1 is additionally wedged *frictionlessly* against the
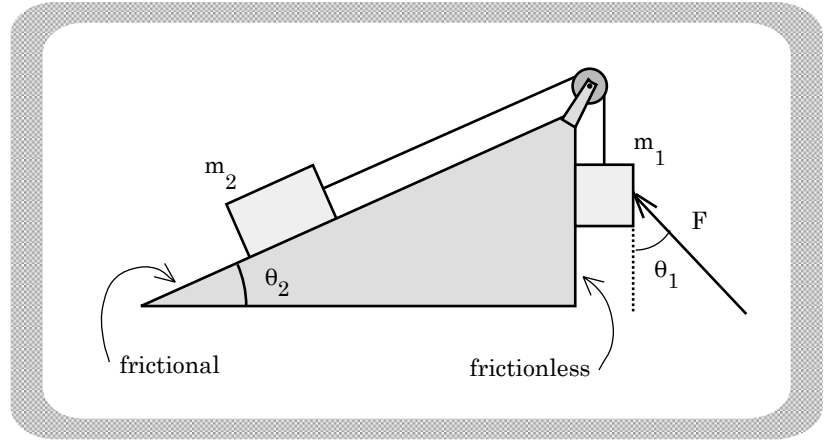

FIGURE 5.12

incline by a known force F (see Figure 5.12). Both the angle of the incline θ_2 and the angle θ_1 at which F assaults the block are known. The *coefficient of kinetic friction* μ_k between m_2 and the incline is also known. Assuming the string is inextensible (that is, it isn't acting like a rubber band) and mass m_2 is moving *up* the incline when first viewed, determine the *magnitude* of the *acceleration* of the system.

Note 1: I've done this analysis in steps to allow you to see how each of the formal steps alluded to in the previous section plays out in a problem. I'm doing it this way ONLY because the approach is new to you and I want you to see and understand what each formal step actually does. At the end of the presentation, I will redo the problem the way I would expect you to proceed if such a problem were given on a test.

Note 2: As the string is inextensible, the "magnitude of the acceleration of the system" will be the *magnitude* of the acceleration of either of the two masses. If we know a for one body, assuming a is a magnitude, we know it for the other.

--Solution and Approach *By-the-Numbers*:

--Step b: The sketch of the system has already been shown in Figure 5.12.

--Step c: We will begin with mass m_1. An f.b.d. of the forces acting on m_1 is shown in Figure 5.13.

Note: The *frictional force* between any two objects sliding relative to one another is $f_k = \mu_k N$. If there had been a *frictional force* acting between m_1 and the incline's vertical wall, we would need to determine N_1 before we could

determine f_k. In this case, there is no frictional force along that surface so there is no need to determine N_1.   Nevertheless, to be complete, our *free body diagram* MUST INCLUDE N_1 and all other horizontal forces acting on m_1.

Bottom line: By definition, a *free body diagram* includes <u>ALL THE FORCES</u> ACTING ON THE BODY FOR WHICH IT IS DRAWN.

FIGURE 5.13

--Step d: Figure 5.14 shows the off-axis-forces broken into their components.

--Step e: The *sum of the forces* in the v *direction* is shown below.

$$\underline{\sum F_v}:$$
$$T - m_1g + F\cos\theta_1 = -m_1(a_v)$$
$$= -m_1a.$$

(*a* is defined to be the <u>*magnitude*</u> of the acceleration of both m_1 and m_2). This implies:

$$T = m_1g - F\cos\theta_1 - m_1a \qquad \text{(Equation A)}.$$

FIGURE 5.14

Note 1: We know that the initial velocity of m_1 is *downward* (m_1 has to be moving downward if m_2 is moving *up* the incline), but we really don't know whether the *acceleration* on m_1 is *up* (i.e., m_1 is slowing) or *down* (i.e., m_1 is increasing speed). Not knowing the relative sizes of m_1 and m_2 makes it impossible to tell which is happening.

As can be seen from the work done above, we are doing this problem on the assumption that the acceleration of m_1 is downward, hence the *negative* sign in front of the m_1a term on the right-hand side of N.S.L. If we are wrong, it won't matter as long as we are *consistent throughout the problem*. That is, if we assume m_1 accelerated *downward,* we must also assume that m_2 accelerates *up the incline.*

As was noted some sections ago, when the mass values are put into the final equation, the *sign* of the calculated acceleration magnitude will either be *positive* or *negative.* If it is positive, it means we have assumed the correct
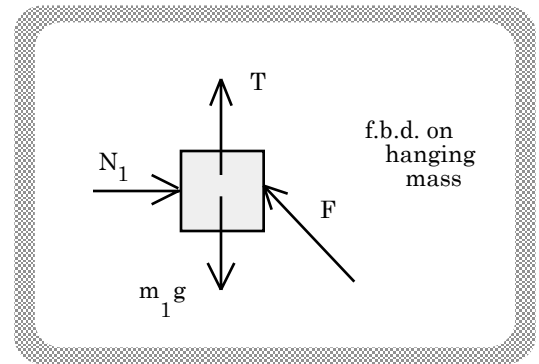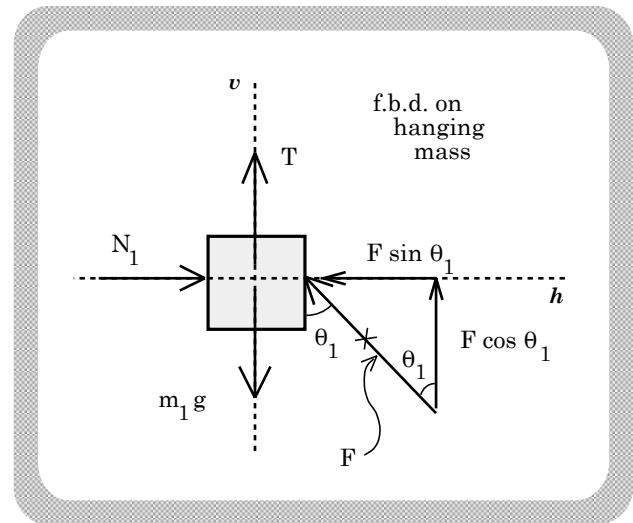
130

direction for the acceleration; if it is negative, it means we have assumed the wrong direction. The magnitude will be the same in either case.

    --Step f: We have one equation (Equation A) and two unknowns (T and a). We need another equation, which means we must look at m_2's motion. Starting the approach over again on m_2:

<u>For m_2:</u>

    --Step b: The sketch of the system is still shown in Figure 5.12.

FIGURE 5.15

    --Step c: A *free body diagram* of the forces acting on m_2 is shown in Figure 5.15. Notice the frictional force acts *down* the incline--opposite the *upward* motion of m_2.

    --Step d: Figure 5.16 shows the placement of appropriate coordinate axes. The *line* of m_2's acceleration is along the incline, so one of the axes (labeled x) is placed along that line with the second axis (labeled y) *perpendicular* to the first.

FIGURE 5.16

    --Step e: Summing the forces in the *y direction* will allow us to solve for N_2:

$$\underline{\Sigma F_y}:$$

$$N_2 - m_2 g (\cos \theta_2) = m_2 (a_y)$$
$$= 0 \qquad \text{(as } a_y = 0\text{)}$$
$$\Rightarrow \quad N_2 = m_2 g (\cos \theta_2) \qquad \text{(Equation B).}$$

Now that we know N_2, we can sum the forces in the *x direction*:

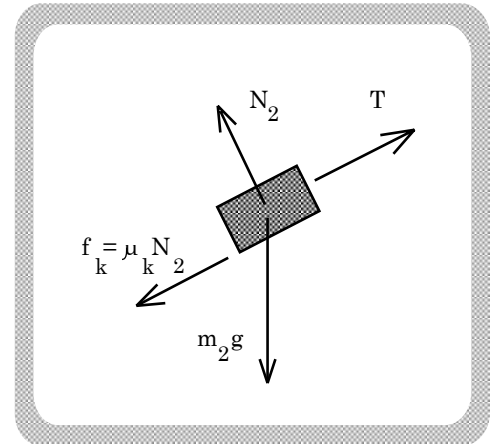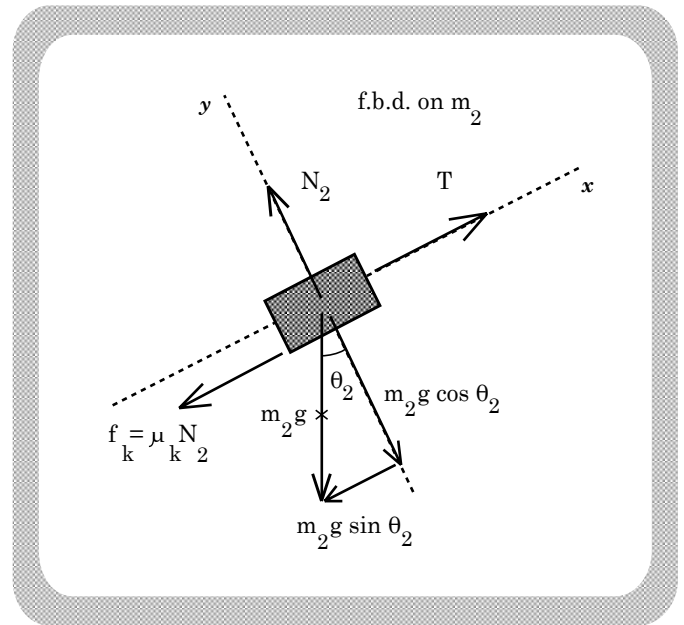$\underline{\Sigma F_x}$:

$$T - m_2g\,(\sin\theta_2) - \mu_k N_2 = m_2(a_x)$$

$$= m_2a,$$

(remember, a was defined as the magnitude of the acceleration of both m_1 and m_2). Substituting *Equation B* in for N_2 yields:

$$T - m_2g(\sin\theta_2) - \mu_k\,[m_2g(\cos\theta_2)] = m_2\,a.$$

Substituting *Equation A* in for T yields:

$$[m_1g - F\cos\theta_1 - m_1a] - m_2g(\sin\theta_2) - \mu_k\,[m_2g(\cos\theta_2)] = m_2a$$

$$\Rightarrow\quad a = [m_1g - F\cos\theta_1 - m_2g(\sin\theta_2) - \mu_k m_2g(\cos\theta_2)] \,/\, (m_1 + m_2).$$

--**Step 6:** Finito!

4.) <u>Example 2: Incline and Pulley using the Formal Approach as expected ON A TEST:</u>

<u>For m_1</u>: (f.b.d., axes, and components shown in Figure 5.14)

$\underline{\Sigma F_v}$:

$$T - m_1g + F\cos\theta_1 = -m_1(a)$$
$$\Rightarrow\quad T = m_1g - F\cos\theta_1 - m_1a \quad \text{(Equation A)}.$$

<u>For m_2</u>: (f.b.d., axes, and components shown in Figure 5.16)

FIGURE 5.14

$\underline{\Sigma F_y}$:

$$N_2 - m_2g\,(\cos\theta_2) = m_2\,(a_y) \qquad (= 0 \text{ as } a_y = 0)$$
$$\Rightarrow\quad N_2 = m_2g\,(\cos\theta_2) \qquad\qquad \text{(Equation B)}.$$
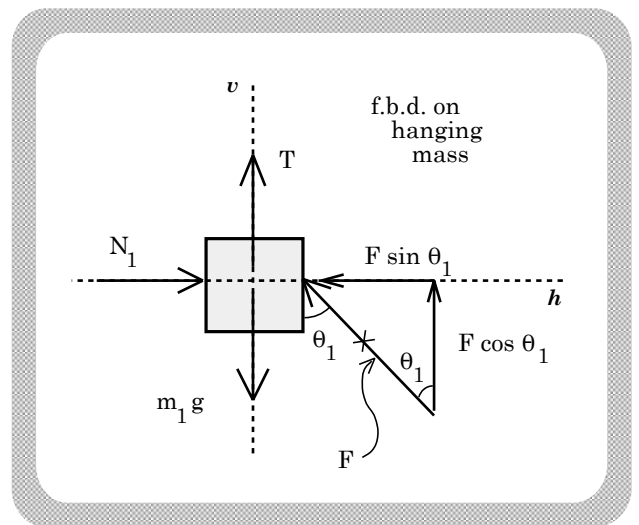
$\sum F_x$:

$T - m_2g (\sin \theta_2) - \mu_k N_2 = m_2(a_x)$

$= m_2a,$

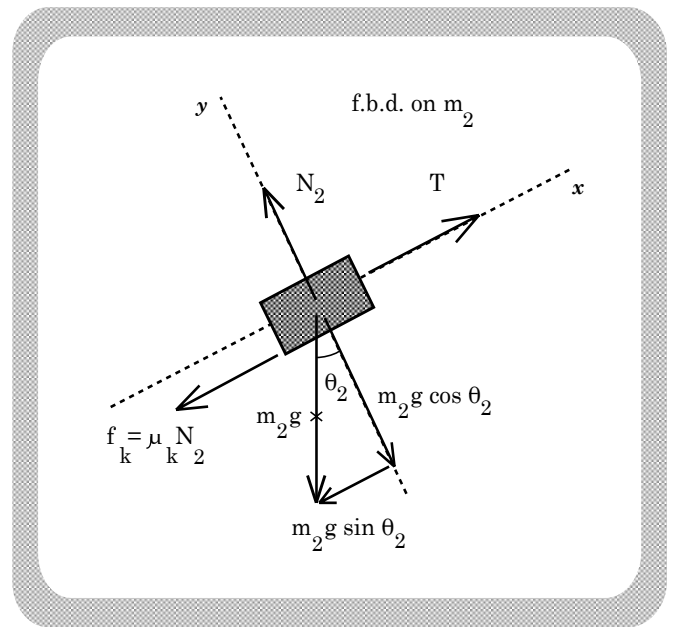Using *Equation B* to eliminate N_2 and *Equation A* to eliminate *T*, we can write:

FIGURE 5.16

$$T \qquad - m_2g(\sin \theta_2) - \mu_k \qquad N_2 \qquad = m_2a$$

$$[m_1g - F\cos \theta_1 - m_1a] - m_2g(\sin \theta_2) - \mu_k [m_2g(\cos \theta_2)] = m_2a$$

$$\Rightarrow \quad a = [m_1g - F\cos \theta_1 - m_2g(\sin \theta_2) - \mu_k m_2g(\cos \theta_2)] / (m_1+ m_2).$$

5.) Observations about *Example Problem 2*:

a.) Let's try some numbers: Assume $m_1 = 5\ kg$, $m_2 = 8\ kg$, $\mu_k = .4$, the applied force $F = 3\ newtons$, $\theta_1 = 50^0$, and $\theta_2 = 30^0$. Re-ordering, remembering that the magnitude of the *acceleration of gravity g = 9.8 m/s^2*, and omitting units to save space, the calculation becomes:

$$a = [[m_1g - m_2g(\sin \theta_2) - \mu_k m_2g(\cos \theta_2)] - F\cos \theta_1] / (m_1+ m_2)$$
$$= [g [m_1 - m_2(\sin \theta_2) - \mu_k m_2(\cos \theta_2)] - F\cos \theta_1] / (m_1+ m_2)$$
$$= [(9.8) [(5) - (8) \sin 30^0 - (.4) (8) \cos 30^0] - (3) \cos 50^0] / (5 + 8)$$
$$= -1.48\ m/s^2.$$

Note: We have a *negative sign* in front of the calculated acceleration. Why? In the problem, the hanging mass m_1 was stated to be traveling downward (that was the way the problem was set up). Additionally, we assumed that m_1 was *accelerating* downward (we didn't know for sure--that was our

guess). What the *negative sign* means is that, given the mass-values used, m_1's *acceleration* was not *downward* but *upward*. Evidently, the hanging mass moved downward, *slowing in the process*.

b.) If m_1 had been moving *upward*, what would have been different? The *free body diagram* for m_1 would have looked identical to that shown in Figure 5.13, but m_2's f.b.d. would have been different (see Figure 5.17). We still wouldn't have known the acceleration's direction, so we still would have had to assume one. If we had taken it to be *up the incline*, the final acceleration expression derived from Newton's Second Law would have been:
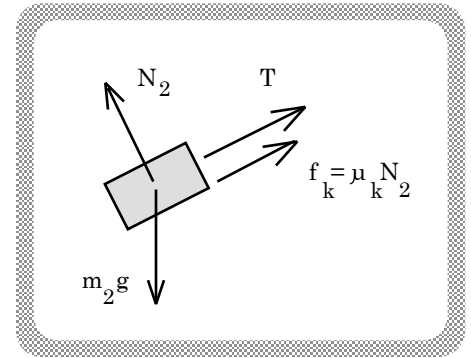
$$a = [[m_1g - m_2g(\sin \theta_2) + \mu_k m_2g(\cos \theta_2)] - F\cos \theta_1] / (m_1 + m_2)$$
$$= [g[m_1 - m_2(\sin \theta_2) + \mu_k m_2(\cos \theta_2)] - F\cos \theta_1] / (m_1 + m_2)$$
$$= [(9.8)[(5) - (8)\sin 30^0 + (.4)(8)\cos 30^0] - (3)\cos 50^0] / (5 + 8)$$
$$= +2.69 \text{ m/s}^2.$$

Notice the acceleration value in this case turns out to be positive, implying that the assumed direction-of-acceleration for m_1 (downward) was correct (given the mass values we've used in the evaluation). Evidently, in this case, the hanging mass moved upward, *slowing in the process*.

Conclusion: In both cases, friction was great enough to slow the motion.

6.) Example 2--Incline and Pulley using the Simple Approach: A string attached to a known hanging mass m_1 is threaded over a *frictionless, massless pulley* and attached at its other end to a second known mass m_2 that is supported by a *frictional* incline plane. The mass m_1 is additionally wedged *frictionlessly* against the incline by a force F (see Figure 5.12). Both the angle of the incline θ_2 and the angle θ_1 at which F assaults the block are known. The *coefficient of kinetic friction* μ_k between m_2 and the incline is also known. Assuming the string is inextensible (that is, it isn't acting like a rubber band) and mass m_2 is moving *up* the incline when first viewed, determine the *magnitude* of the *acceleration* of the system.

a.) As before, the question to ask is, "What are the acting forces that motivate the system as a whole to accelerate?"

b.) In this case, there are several forces motivating the total mass of the system (i.e., $m_1 + m_2$) to accelerate. They are:

i.) The force of gravity on m_1: Its magnitude is $m_1 g$ and its effect via the string is to attempt to accelerate m_2 UP the incline.

Note: If we are talking about gravity on m_1, why do we care about m_2? We need some way to assign positiveness or negativeness to forces acting on *the SYSTEM* when, in fact, those forces may well be acting on different bodies oriented in entirely different ways from one another. The easiest way to do that is to pick a single body in the system and determine how a given force will ultimately affect that body. In this case, gravity is pulling m_1 down. This will tend to pull m_2 UP the incline, so I will arbitrarily identify that effect to be associated with a positive force.

ii.) The component of F that pushes m_1 upward: Its magnitude will be $F \cos \theta_1$. Its effect is to attempt to accelerate m_2 DOWN the incline, so we will call it a negative force.

iii.) The component of gravity that pushes m_2 DOWN the incline: Its magnitude is $m_2 g \sin \theta_2$ (this is the component of gravity *along the line of the incline*), and we will call it a negative force.

iv.) The force of friction on m_2: With the normal force equal to $m_2 g \cos \theta_2$, friction's magnitude is $f_k = m_k N = m_k(m_2 g \cos \theta_2)$. The direction of the frictional force will be opposite the direction of motion, or DOWN the incline, making it a negative force.

v.) The temptation might be to add in the tension force. In this case, tension is an internal force--a force that exists as a consequence of the interaction of pieces of the system. Internal forces will not affect the overall motion of the system. That is, if we add the tension force on m_1 (this will be upward on m_1 motivating m_2 to accelerate down the incline) to the tension force on m_2 (this will motivate m_2 to accelerate up the incline), the net effect will be zero. As such, tension IN THIS CASE is ignored.

c.) Summing up all of the forces and putting them equal to the total mass in the system, we get:

$$m_1 g - F(\cos \theta_1) - m_2 g(\sin \theta_2) - \mu_k(m_2 g \cos \theta_2) = (m_1 + m_2)a$$
$$\Rightarrow \quad a = [m_1 g - F\cos \theta_1 - m_2 g(\sin \theta_2) - \mu_k m_2 g(\cos \theta_2)] / (m_1 + m_2).$$

Note: AGAIN, don't get too excited about all of this. The simplified approach isn't always going to work. Get to know both procedures!

E.) Centripetal Force:

1.) A net force acting on a body produces an acceleration, and an acceleration implies a *change of velocity*. So far, the only accelerations we have dealt with have been those that change the *magnitude* of a body's velocity--that make objects *speed up* or *slow down*. There is another way an acceleration can change a body's motion: it can change the <u>direction</u> of the body's *velocity* vector.

2.) Example--the M.O.B. maneuver:

a.) In days of yore when rock-n-roll was in and the '55 Chevy was the *hot rod* of choice, there was a technique the execution of which guaranteed, if not romance, at least close contact between a guy and his date. It was called the infamous M.O.B. maneuver.

<u>The Situation:</u> A boy picks up his date (we will assume the boy is driving). The two like one another. The vehicle is a 1955 Chevy without seat-belts (the norm for 1950's cars). It sports a bench seat (also the norm for 1950's cars). She gets in and sits next to the passenger-side door. He'd like her to sit next to him but is too bashful to ask her to move over. She'd *like* to sit next to him but is too shy to do it on her own.

Solution: the M.O.B. maneuver.

<u>The Maneuver:</u> Traveling at 30 miles per hour, the boy casually approaches an appropriate intersection (see Figure 5.19).

Upon reaching the corner, he makes a hard right turn without warning.

Consequence: she finds herself seated next to him in short order.

How so?

He is attached to the turning car via the steering wheel. As the car turns, the steering wheel exerts a force on him that makes him move on the *same semi-circular path* the car follows. She, on the other hand, is *not* attached to the car (the maneuver works best if the seat has been waxed beforehand). As there are no forces acting on her, she does exactly what any force-free body will do--she continues to move in a *straight line* (Newton's First Law--bodies in motion tend to stay in motion in a straight line unless impinged upon by a force). Doing so insures that he and she will, indeed, meet somewhere around the BINGO in Figure 5.20.

intersection

initial direction
of motion

'55 Chevy

FIGURE 5.19

projected force-free
path of the girl

BINGO

path of car
and boy

FIGURE 5.20

Note: You might wonder why it is called an MOB maneuver? Although it is probably *politically* and *socially* improper today, in the 1960's it stood for *Move Over, Baby*.

b.) The Point: An acceleration, hence a net force, is needed to *change the direction of a moving body* (hence, the *direction* of the body's *velocity* vector). That force will be perpendicular to the direction-of-motion, which is to say perpendicular to the direction of the velocity vector.

Note 1: The boy entered the curve at 30 mph, and he left the curve at 30 mph. The *magnitude* of his velocity didn't change, but the *direction* did. That would not have been the case if the force provided by the steering wheel had been in any direction other than *perpendicular to the boy's path*.
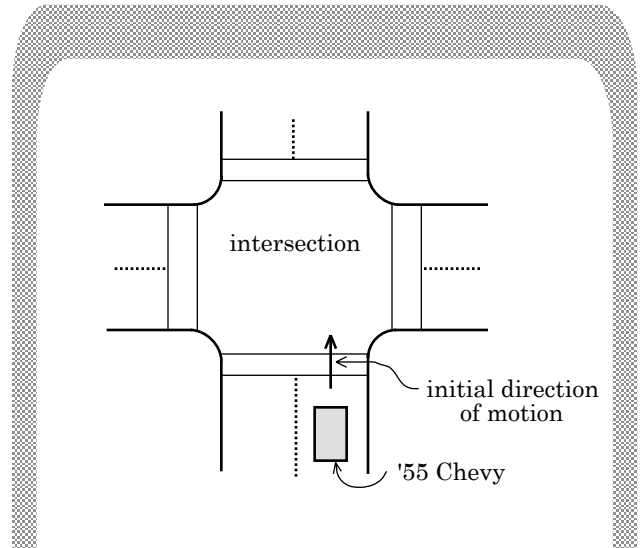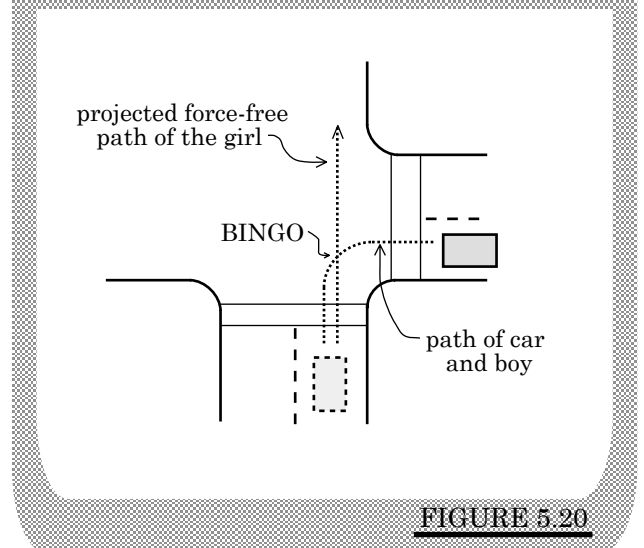
Note 2: Assuming this direction-changing force is constant in magnitude, it will always make the body move along a circular path about a fixed center. Forces that do this are often referred to as "center-seeking forces."

Note 3: The phrase *center-seeking* is a label only. In a given situation, the combination of forces that collectively qualify for that moniker must exist naturally within the system. They can be *normal* forces, *tension* forces, *gravitational* forces, *frictional* forces, *push-me-pull-you* forces, or some combination thereof.

Note 4: The word "centripetal" means *center-seeking*. It is the <u>combination of forces that are center-seeking</u> that is labeled the net *centripetal force*. Again, this is a label only. We are not talking about a new *kind* of force--only a new kind of *situation* in which the same old forces might be applied.

Note 5: Students often become confused when dealing with *centripetal forces* generated by friction between a car's tires and the road. For clarification, consider the car example above with one modification: assume the car is driving on a *dirt* road. What happens when the car's wheels are cranked into a hard, tight turn (see Figure 5.21)?

The car's tires apply a *frictional force* to the ROAD and end up throwing dirt outward away from the car's ultimate path. The force the *car applies to the road* is, therefore, <u>directed outward</u> relative to the ultimate path of the car. On the other hand, the *road* applies a frictional force f_{car} to the car which, according to Newton's Third Law, is *equal in magnitude* and *opposite in direction* to the force the car applies to the road.

direction
of force
*car applies
to dirt road*
(a centri-
fugal force)

v

force component
road applies to car
in center seeking
(centripetal) direction

force component
road applies to car
opposite motion
(tends to slow car)

car wheel
as viewed
from above

direction of force
road applies to car

FIGURE 5.21

This force, f_{car}, has one component that is oriented opposite the direction of the car's motion and another component that is perpendicular to the direction of motion. The latter component pushes the CAR inward, making it follow a curved path. This is the center-seeking (*centripetal*) force in the system.
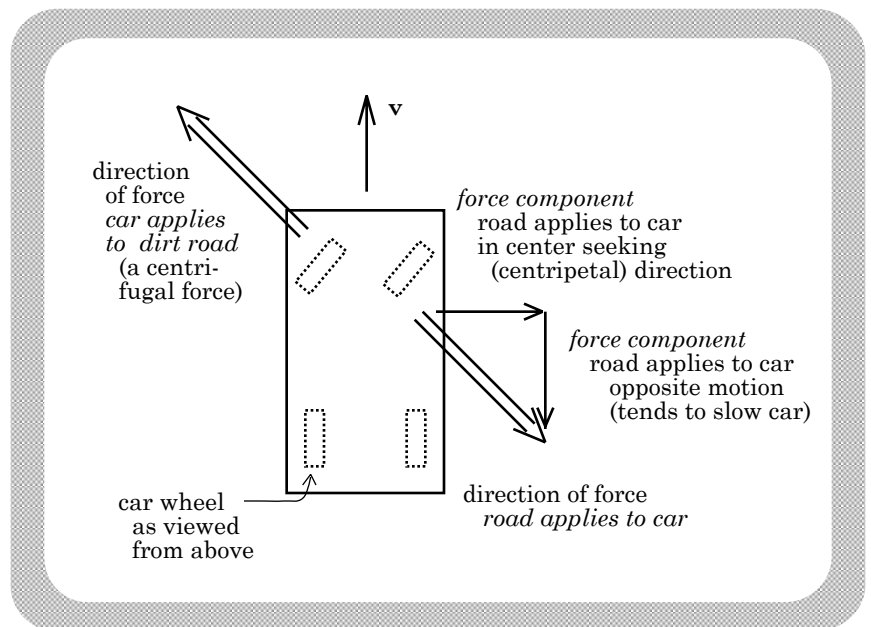
When using Newton's Second Law to analyze the motion of the car, the only forces we are interested in are the forces *acting on the car*. As such, the force the car applies to the road has no place in the analysis.

3.) Other Examples:

a.) Consider the moon orbiting the earth: The moon does not move off into space in a straight line because the earth exerts a *gravitational force* on the moon, pulling it away from the path it would have taken if it had been force-free (see Figure 5.22). Consequence: the moon is pulled into a nearly circular path around the earth.

In this case, *gravity* is the *center-seeking* (centripetal) force that naturally exists within the system.

earth

actual path of moon

direction of gravitational force (a centripetal force in this case) exerted by earth on moon

moon

force-free direction moon would take without applied centripetal force
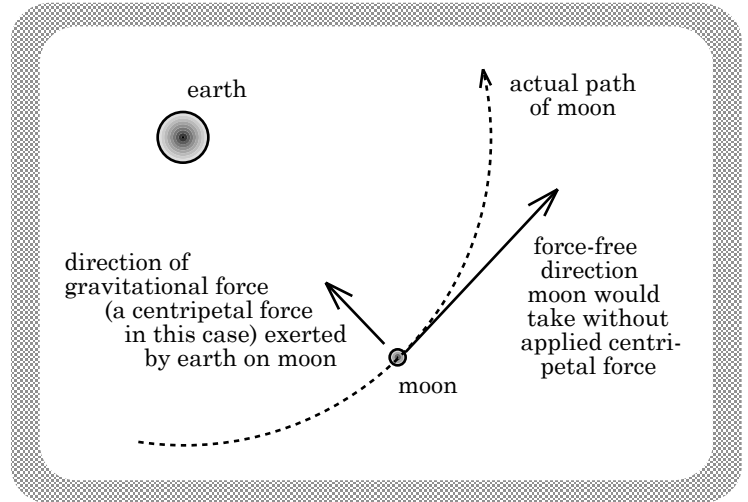
FIGURE 5.22

Note: The moon's path isn't perfectly circular due to the initial conditions under which the moon's motion was generated.

b.) The *hammer throw* in Olympic competition is executed using a mass (a ball) attached to a chain. The contestant grasps the chain, swings it around and around until he or she reaches a relatively high spin-speed, and then lets the ball and chain go. The proper technique does not suggest twirling the ball overhead in a horizontal plane, but let's assume old Joe is a bit off-form and does not know any better (see Figure 5.23). From

component of chain force that counteracts gravity

chain

center-seeking component of chain force on ball (note that the radius of the motion is not the length of the chain)
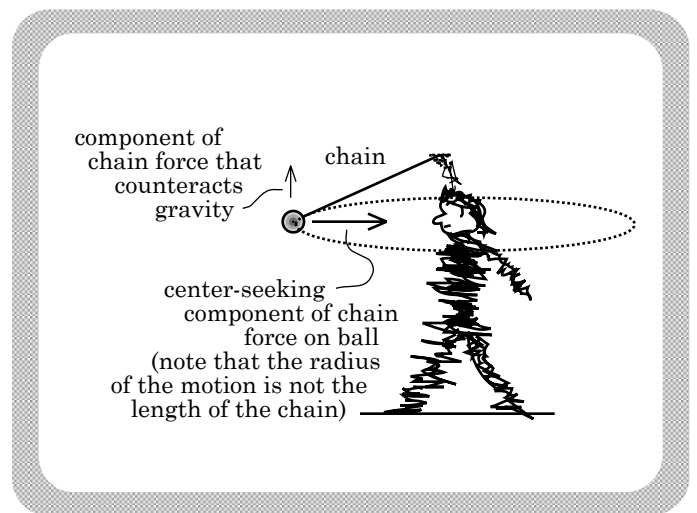
FIGURE 5.23

whence does the *center-seeking* (centripetal) force on the ball come?

It comes from the *horizontal component* of the *tension* force provided by the chain. That component is the *centripetal force* in the system.

4.) Example Problem--<u>Deadman's Curve</u>: A car approaches a curve of known radius R. The *coefficient of static friction* between the tires and the road is μ_s. At what *maximum velocity* can the car take the curve without breaking traction and spinning out (Figure 5.24 shows situation from above)?

Note: Assume the wheels are turned so slightly that the component of f_s that opposes the car's motion is zero. That is, assume f_s is fully centripetal.

a.) This is a Newton's Second Law problem. We need to identify the important forces acting on the car, so we will use an f.b.d. perspective that views the car head-on (the view shown in Figure 5.25 depicts that perspective).

FIGURE 5.24

b.) For the vehicle to follow a circular path, we know there must be a force, hence acceleration, in the center-seeking direction (i.e., in the direction oriented *toward the <u>center of the circle upon which the car is moving</u>*).

Note: The *centripetal direction* will be *perpendicular* to the direction of motion. As seen in our sketch, the car is coming *out of the page* which means the center-seeking direction is to the right in the plane of the page.
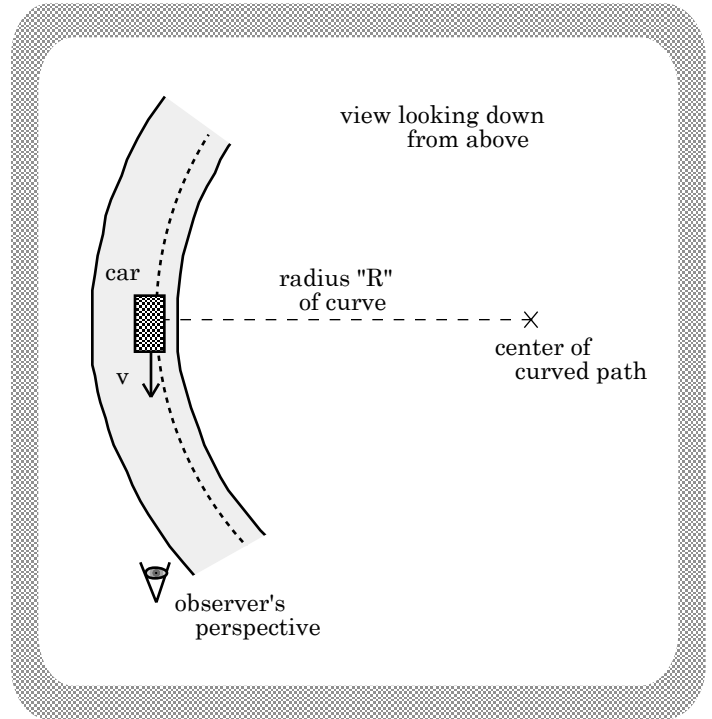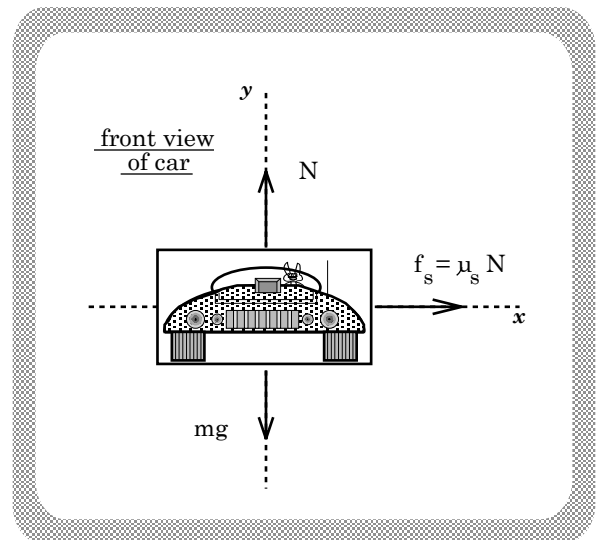

FIGURE 5.25

c.) Using N.S.L., we get:

$$\underline{\sum F_y}:$$

$$N - mg = ma_y.$$

As $a_y = 0$ (the car is not jumping up off the road), we get:

$$N = mg.$$

$$\underline{\sum F_{\text{center-seeking}}}:$$

$$\mu_s N = ma_c.$$

Substituting in $N = mg$ from above and solving for a_c, we get

$$a_c = \mu_s (mg)/m$$
$$= \mu_s g.$$

Note: This will be the maximum acceleration the road/tire contact can generate. How so? Because $\mu_s N$ is not just any static frictional force, it is the *maximum* static frictional force. It will be related to the maximum *centripetal acceleration* available to the car when it attempts to follow the curve, which in turn is related to the maximum *velocity* possible.

d.) If we had a constant acceleration whose direction was *tangent to the curve*, the velocity's magnitude would change with time and we could determine *v(t)* by solving the expression $a = dv/dt$. The problem? The acceleration a_c is *radially* directed. That means it is not oriented to change the velocity's magnitude--it is oriented to change the velocity's direction. As such, neither $a = dv/dt$ nor kinematics will help us.

We need to derive a relationship between the *velocity of the car* and the *acceleration* component that is changing the car's *direction* of motion (i.e., the centripetal acceleration). That derivation is at the end of the chapter (you'll never be asked to reproduce that derivation by yourself--it has been included so that you can convince yourself that nothing *dirty* has been done here).

e.) <u>The bottom line of that derivation:</u> if a body moving with *velocity magnitude v* is to move in a circular path of radius R, it must experience a center-seeking (centripetal) acceleration a_c numerically equal to v^2/R.

f.) With this information, we return to our problem. If $a_c = v^2/R$ and, from above,

$$a_c = \mu_s g,$$

we get

$$v^2/R = \mu_s g$$
$$\Rightarrow \quad v = (\mu_s R g)^{1/2}.$$

Note 1: This solution does make sense intuitively. Think about your own experience on the highways and byways of America: tight turns are taken slowly while wide turns are taken at higher speeds. That is exactly what our equation predicts. The larger the curve (i.e., the bigger R), the larger the predicted maximum velocity.

Note 2: The units for the variables under the radical had better work out to *meters per second.* Checking: the coefficient of static friction is unitless; R has the units of *meters*; and g the units of *meters per second squared.* The net units under the radical are *meters squared per seconds squared*, the square root of which is *meters per second.*

5.) Example Problem 2--<u>Deadman's Curve with Banked Incline:</u> Our car now approaches a banked curve of angle θ, radius R, and coefficient of static friction μ_s. What is the maximum velocity v_{max} with which the car can take the curve without flying up over the top of the embankment?

Note 1: As viewed from above, the car moves in a semi-circular path that looks exactly like the path taken by the car in the previous problem. We will, therefore, use Figure 5.24 to depict the overhead view here. A head-on perspective is shown in Figure 5.26. A *free body diagram* for the situation is shown in Figure 5.27.

Note 2: The *maximum velocity* will occur when the maximum static frictional force just holds the car from breaking loose and sliding up over the top of the bank. That frictional force must, therefore, be directed down the incline.
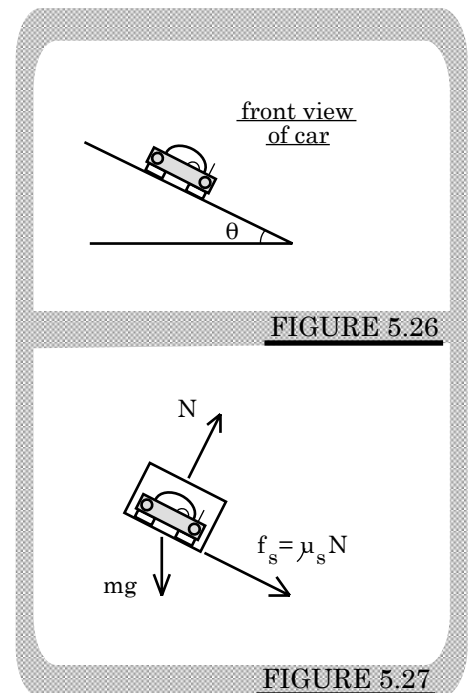

front view
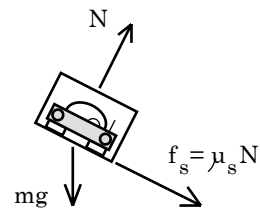of car

FIGURE 5.26

N

mg

$f_s = \mu_s N$

FIGURE 5.27

Note 3: One axis must be along the line of centripetal acceleration. That *center-seeking direction* is oriented along the line of the RADIUS of the car's motion. That direction is *not* along the line of the incline. That is, even though the car is physically tilted, the direction toward the CENTER of the car's circular path is in the *horizontal*. As such, one of the axes has been defined in that direction.

a.) Examining the modified *free body diagram* (modified in the sense that axes and force components have been included) shown in Figure 5.28, Newton's Second Law yields:
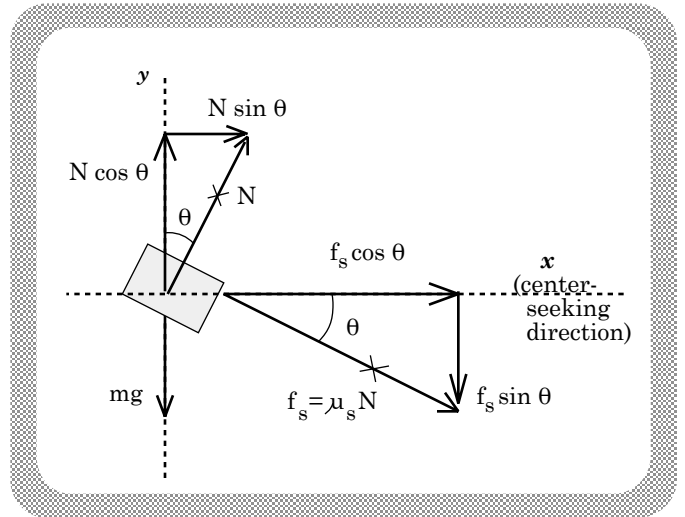
FIGURE 5.28

$$\underline{\Sigma F_y}:$$
$$N \cos \theta - mg - f_s \sin \theta = ma_y.$$

b.) As $a_y = 0$, and noting that $f_s = \mu_s N$, we get:

$$N \cos \theta - mg - (\mu_s N) \sin \theta = 0$$
$$\Rightarrow \quad N = (mg) / (\cos \theta - \mu_s \sin \theta).$$

c.) In the *center-seeking* direction:

$$\underline{\Sigma F_{center-seeking}}:$$
$$N \sin \theta + f_s \cos \theta = ma_c.$$

d.) Substituting in $f_s = \mu_s N$, $N = [(mg)/(\cos \theta - \mu_s \sin \theta)]$, and letting $a_c = v^2/R$, the equation

$$N \sin \theta + \mu_s N \cos \theta = ma_c$$

becomes

$$[(mg)/(\cos \theta - \mu_s \sin \theta)]\sin \theta + \mu_s[(mg)/(\cos \theta - \mu_s \sin \theta)] \cos \theta = mv^2/R.$$

e.) Dividing out the *m's* and rearranging, we get:

$$v_{max} = [R\ (g\ \sin\ \theta + \mu_s g\ \cos\theta)/(\cos\theta - \mu_s \sin\theta)]^{1/2}.$$

Note 1: When $\theta = 0$ (i.e., the same situation we had in the Example Problem 1), we get $v_{max} = (\mu_s Rg)^{1/2}$ as expected.

Note 2: For $\theta = 30^0$, $\mu_s = .25$ (this corresponds to a relatively slick roadway), and a radius of 50 meters, the calculated value of $v_{max} = 21.77$ m/s. This is right around 50 mph.

6.) Alternate Example-- Problem 2: The *free body diagram* in Figure 5.29 presents another possibility. If the angle of the inclined road is great enough, it is possible that the maximum static frictional force on a stationary or slowly moving car would not be large enough to hold the car on the incline. Consequence: the car would break loose and slide *down* the incline.

In that case, a car could make it through the turn but it would have to be moving with a velocity above some minimum value. As the car would want to slide down the incline

FIGURE 5.29

for velocities below that minimum, the static frictional force would be up the incline (the car wants to slide down).

Consider the car and embankment used in *Part 5* above. What is the *minimum velocity* v_{min} the car can take the curve without sliding down into the infield at the bottom of the embankment?

a.) The f.b.d. for this situation is shown in Figure 5.29:

b.) N.S.L. maintains:

$\underline{\Sigma F_y}$:
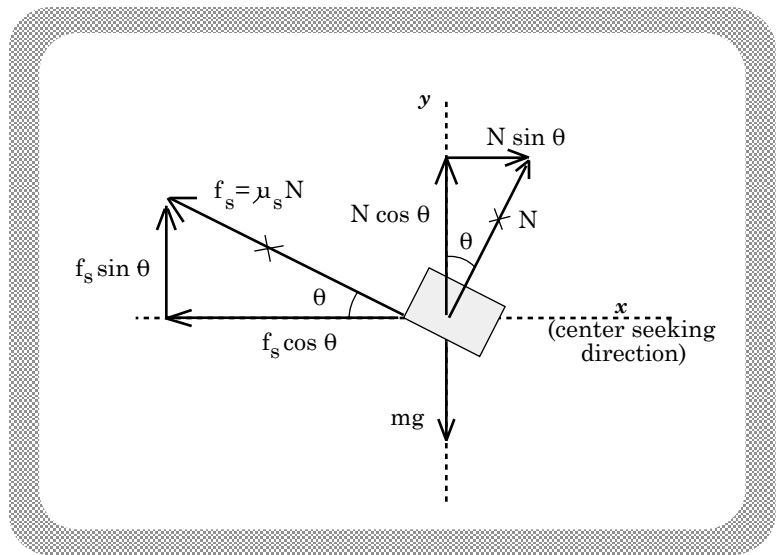
$$N \cos\ \theta - mg + f_s\ \sin\ \theta = ma_y.$$

As $a_y = 0$ and $f_s = \mu_s N$, we get:

$$N \cos \theta - mg + (\mu_s N) \sin \theta = 0$$
$$\Rightarrow \quad N = (mg) / (\cos \theta + \mu_s \sin \theta).$$

c.) In the *center-seeking* direction:

$$\underline{\Sigma F_{center\text{-}seeking}}:$$

$$N \sin \theta - f_s \cos \theta = ma_c.$$

d.) Substituting $f_s = \mu_s N$; substituting the quantity *[(mg)/(cos θ + $m_s$sin θ)]* for *N*; and setting $a_c = v^2/R$, the equation

$$N \sin \theta - \mu_s N \cos \theta = ma_c$$

becomes

$$[(mg)/(\cos\theta + \mu_s\sin\theta)]\sin\theta - \mu_s[(mg)/(\cos\theta + \mu_s\sin\theta)]\cos\theta = mv^2/R.$$

e.) Dividing out the *m's* and rearranging, we get:

$$v_{min} = [R (g \sin \theta - \mu_s g \cos \theta)/(\cos \theta + \mu_s \sin \theta)]^{1/2}.$$

Note 1: When $\theta = 0$ (the situation we had in the Example Problem 1), we get:

$$v_{min} = (-\mu_s Rg)^{1/2},$$

an imaginary value. This implies that there is *no minimum speed* at which a car could take a flat curve and *not* make it through.

Note 2: For $\theta = 30^0$, $\mu_s = .25$ (again, this is a very slick roadway), and a radius of 50 meters, the calculated value of $v_{min} = 11.9 \ m/s$. This is around 25 mph. Below this speed, the car will not hold traction with the road and will slide down toward the bottom of the bank.

7.) <u>Derivation of Centripetal Acceleration Expression</u>: You will not be asked to duplicate this proof; it is included for the sake of completeness.

It was stated above that the magnitude of any *centripetal acceleration* acting on an object moving in a circular path is numerically equal to v^2/R, where v is the magnitude of the object's velocity and R the radius of the path. The derivation from which that claim comes follows:

<u>FIGURE 5.30a</u>

a.) A body with a constant velocity-magnitude v moves on a circular path of radius R as shown in Figure 5.30a. Between times t_1 and t_2, the body moves through an angular displacement of $\Delta\theta$. Figure 5.30b shows this set-up along with a secant connecting the position points at times t_1 and t_2. Notice that:

<u>FIGURE 5.30b</u>

i.) The triangle is isosceles (two of the sides are equal in length to the radius R of the motion);

ii.) The third side of the triangle (the secant) is identified as Δs;

iii.) Δs and the arc length subtended by $\Delta\theta$ are not equal as long as $\Delta\theta$ is large; and

iv.) The arc length is numerically equal to the velocity-magnitude v times the time interval (Δt) required to travel between positions 1 and 2, or:
$$\text{arc length} = v\,\Delta t.$$

b.) The vector difference between the velocity vectors at t_1 and t_2 yields a change of velocity vector Δv over that period of time. That vector subtraction is shown in Figure 5.31. Notice that:
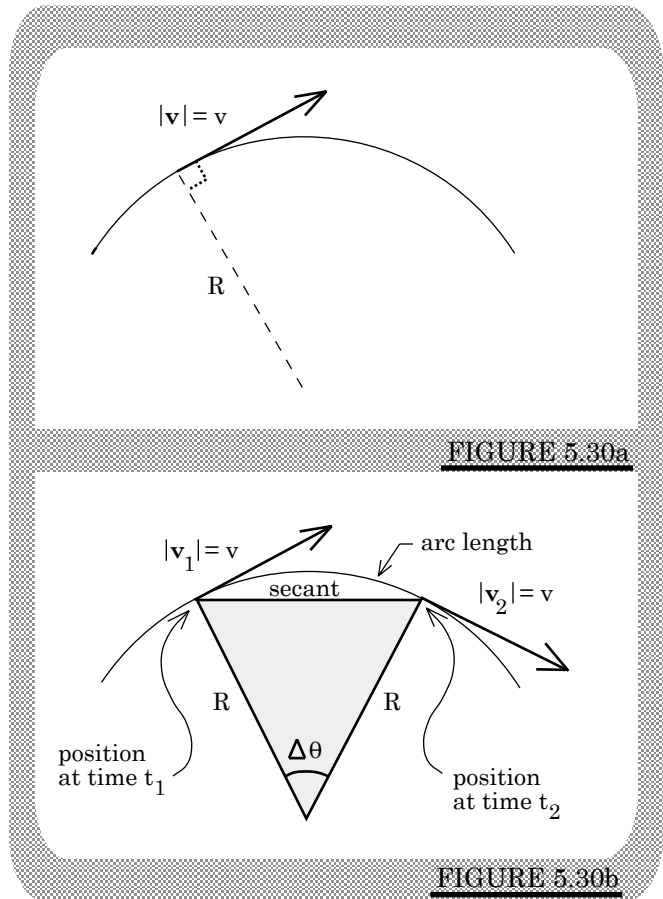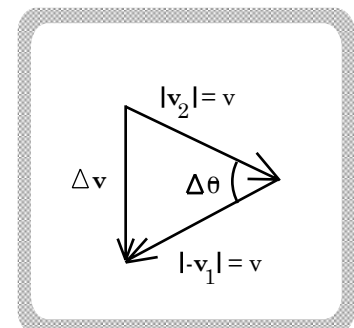

<u>FIGURE 5.31</u>

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i.) The direction of the vector difference Δv is approximately oriented toward the center of the circular path on which the body moves;

ii.) The triangle created by the subtraction is isosceles (two of the sides are numerically equal to the magnitude of the constant velocity v); and

iii.) Because the velocity vectors are perpendicular to their respective radii, the isosceles triangle created by the radii and secant (Figure 5.30b) is *similar* to the triangle created by the velocity-vector-difference (Figure 5.31).

c.) With *similar* triangles, the ratio of any two sides of one triangle will be equal to the ratio of the corresponding sides of the second triangle. As such, we can write:

$$\Delta v/v = \Delta s/R.$$

d.) If the time interval is allowed to become very small (i.e., t_1 and t_2 approach one another), the arc length approaches Δs and we can write:

$$\text{arc length} = \Delta s \text{ (in the limit as } \Delta t \text{ goes to zero).}$$

But the arc length equals $v\Delta t$, which implies that as $\Delta t \rightarrow 0$,

$$\Delta s \rightarrow v\Delta t.$$

e.) Plugging this back into our ratio, we can re-write $\Delta v/v = \Delta s/R$ as:

$$\lim_{\Delta t \rightarrow 0}\left(\frac{\Delta v}{v}\right) = \left(\frac{v\Delta t}{R}\right).$$

Rearranging yields:

$$\lim_{\Delta t \rightarrow 0}\left(\frac{\Delta v}{\Delta t}\right) = \left(\frac{v^2}{R}\right).$$

f.) The left-hand side of this expression is the definition of instantaneous acceleration. What acceleration? The acceleration whose direction is *center-seeking*.

Bottom line: The *magnitude* of the *centripetal acceleration* required to move a body whose velocity magnitude is v into a circular path of radius R is v^2/R.

F.) A Note About MASS:

1.) Although the material you are about to read will not be included on a test, it is important that you understand what *mass* really is. READ THIS SECTION ONCE. If nothing else, it will help you stave off massive confusion later on when we get to rotational motion.

2.) There are certain characteristics that are true of all material objects. For instance, all objects have *a tendency to resist changes in their motion.*

a.) Example: A rock placed in space will not suddenly, sponta-neously accelerate for no reason. It will sit in its place until a force makes it move.

b.) The unwillingness of an object to spontaneously change its motion is called *inertia.* As the amount of inertia an object has is intimately related to how much *force* will be required to *accelerate* the object, quantifying the idea of inertia is important. Early scientists satisfied that need by defining an *inertia-related quantity* they called "inertial mass."

c.) The system they devised is simple. A platinum-iridium alloy cylinder, currently housed in a vault at the *Bureau of Weights and Measures* in Sevres near Paris, France, was defined as having *one kilogram* of inertial mass. All other inertial mass values are measured relative to that cylinder. That is, an object with the same amount of *resistance to changing its motion* as does the standard is said to have "one kilogram of *inertial mass.*" An object with twice the *resistance to changing its motion* is said to have two kilograms of *inertial mass*; one-half the resistance implies one-half kilogram of *inertial mass,* etc.

In other words, the inertial mass of a body gives us a numerical way of defining how much inertia an object has RELATIVE TO THE STANDARD. Put still another way, *inertial mass is a <u>relative measure</u> of a body's tendency to resist changes in its motion.*

d.) Although France is a beautiful country, it would be terribly inconve-nient for laboratory scientists around the world if they had to travel to France every time they wanted to determine an *inertial mass* value, so scientists fur-ther generated a laboratory technique for measuring *inertial masses*. It uti-

lizes an inertial balance--a tray mounted on two thin blades that allow the tray to vibrate back and forth. The more mass that is placed in the tray, the slower the tray vibrates. A simple formula relates the tray's vibratory rate (its *period* of motion) to the amount of *inertial mass* there is in the tray.

Although it works, using an inertial balance is a VERY CUMBER-SOME and time consuming operation.

3.) There are other characteristics that are true of material objects. For instance, all massive objects are attracted to all other massive objects (at least according to Newton). We call this attraction *gravity*.

   a.) A measure of a body's *willingness to be attracted to another body* is related to what is called the "gravitational mass" of an object.

   b.) To provide a quantitative measure of *gravitational mass*, scientists have taken an agreed upon object as the standard against which all subsequent *gravitational mass measurements* are made (again, this standard is housed today in Sevres, France).

   c.) The technique for measuring *gravitational mass* utilizes a *balance* or *electronic scale*. The object is placed on a scale which consists of a spring-mounted pan. The gravitational attraction between *the object and the earth* pulls the object toward the earth and compresses the spring in the process. The scale is calibrated to translate spring-compression into gravitational mass (assuming that is what the scale is calibrated to read--in some cases, such scales are calibrated to read force, hence American bathroom scales measure in pounds).
   MEASURING GRAVITATIONAL MASS IS EASY.

4.) Somewhere down the line someone noticed a wholly unexpected and profoundly improbable relationship between gravitational and inertial mass. It was observed that if the same standard (that is, the same object) was used for both, a second object with twice the *gravitational mass* relative to the standard would also have twice the *inertial mass*.

   a.) THIS DOES NOT HAVE TO BE THE CASE. There is no obvious reason why a body with twice the *resistance to changing its motion* (relative to the standard) should also have twice the *willingness to be attracted to other objects*. The two characteristics are completely independent of one another, yet they appear to parallel one another to a high degree of precision (in fact, the best comparisons to date have accuracy to around 10^9 with *no discrepancy found* even at that order of magnitude).

b.) Scientists could have called the units of *gravitational mass* anything they wanted, but because they knew the parallel between *gravitational mass* and *inertial mass* existed, they decided to give *gravitational mass* the units of "kilograms."

That means that as defined, a body with two kilograms of *gravitational mass* also has two kilograms of *inertial mass*.

c.) The beauty of this choice is obvious. Newton's Second Law relates a body's acceleration to the force applied to it. The proportionality constant--the mass term--is an *inertial mass* quantity (the body's resistance to changing its motion is the characteristic that governs how much acceleration the body will feel when a given force is applied). Determining *inertial masses* in the lab is a pain in the arse--inertial balances are not easy devices to set up or use. But *gravitational mass* is easy to measure. All you need is an electronic balance. So when you or I or any lab technician needs to know a body's *inertial mass*, all we have to do is measure the body's gravitational mass on a scale and we have what we want.

d.) <u>Bottom line</u>: Due to the parallel, most people no longer distinguish between *gravitational mass* and *inertial mass*. As the two are numerically interchangeable, people nowadays simply refer to a body's "mass" and leave it at that.

5.) There is an interesting consequence of this peculiar parallel between *gravitational mass* and *inertial mass* which is most easily observed by considering the following question: If a body with two kilograms of *gravitational mass* is attracted to the earth twice as much as a body with only one kilogram of gravitational mass, why doesn't the two kilogram mass free fall toward the earth faster than the one kilogram mass?

a.) The answer is simple. A body with twice the *gravitational mass* also has twice the *inertial mass*; it needs a greater force to overcome its greater inertia.

b.) In short, it does not matter how massive an object is, its *inertia* and its *willingness to be attracted to the earth* will always balance one another out making the object accelerate at the same rate as all other objects (assuming you ignore air friction, etc.).

G.) Fictitious Forces: Centrifugal And Others:

1.) Although the phrases look and sound similar, the ideas behind *centripetal force* and *centrifugal force* are patently different.

a.) The phrase *centripetal force* is a label used to identify forces that motivate a body out of straight line motion and into curved motion. These *center-seeking forces* must exist *naturally* within a system.
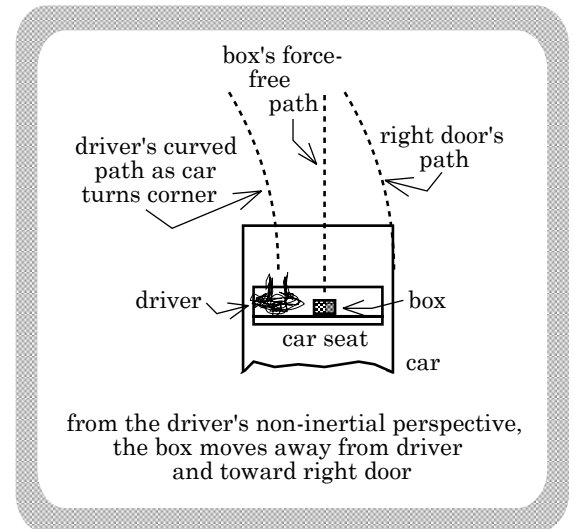
b.) The phrase *centrifugal force* is a label used to identify forces that must be ASSUMED TO EXIST if one is to use Newton's Second Law in analyzing problems from a non-inertial (i.e., accelerated) frame of reference. What does this mean? Read on.

2.) Consider the following example (a slight modification of the M.O.B. maneuver): You are driving in your car. You notice that the box sitting next to you slides *away from you* as you make a left turn (Figure 5.32). Does that mean there is a force acting to push the parcel toward the right door?

box's force-free path

driver's curved path as car turns corner

right door's path

driver

box

car seat

car

from the driver's non-inertial perspective, the box moves away from driver and toward right door

FIGURE 5.32

a.) The answer to the question is *no*. The only forces that are acting on the parcel are gravity and a normal force from the seat (we'll ignore friction). So what is going on?

b.) What is really happening is that *you* are moving away from the parcel (it continues moving in a straight line path, relative to the street) as you and the car are centripetally forced into the turn.

c.) The problem? If you wanted to analyze the parcel's motion from *your frame of reference* (i.e., from a frame attached to the centripetally accelerating car), Newton's Second Law shouldn't work. Why? Because N.S.L. only works in non-accelerated frames of reference.

d.) What's nice is that you *can* make N.S.L. work if you are clever. If you assume there exists a *fictitious force* acting to push the parcel away from you, Newton's Laws will work just fine. For circular motion, that force is called a *centrifugal force*. Its magnitude is equal to mv^2/R. In short, you don't *have* to analyze the problem from a non-inertial frame of reference, but you can if you are so disposed.

3.) After all that, reconsider the initial statement in *Part G-1a* and *b*:

a.) The phrase *centripetal force* is a label used to identify forces that motivate a body out of straight line motion and into curved motion. These *center-seeking forces* must exist *naturally* within a system.

b.) The phrase *centrifugal force* is a label used to identify forces that must be ASSUMED TO EXIST if one is to use Newton's Second Law in analyzing curved-path problems from a non-inertial (i.e., accelerated) frame of reference.

Big Note: What you are about to see is *for your own edification only.* You will not be tested on it. It has been included because the idea of fictitious forces is used a lot at the college level in analyzing certain types of physics problems. If you ever need to understand the approach, the following should help.

4.) You are tethered to the center of a rotating platform (Figure 5.33a). You feel a frictional force that keeps you from sliding laterally across the platform, you feel gravity, you feel a normal force, you feel the force of the tether pulling you in, and you feel what appears to be a force pushing you outward. Let's assume you've measured the tension in the tether and found it to be T. How fast is the platform moving? Analyze this situation from both an inertial and a non-inertial frame of reference.

you sitting on rotating platform
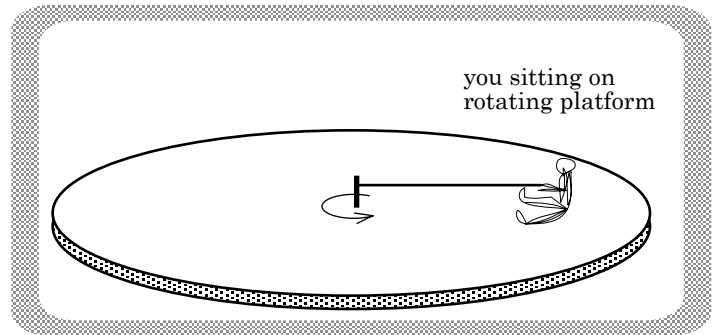
FIGURE 5.33a

a.) From an inertial frame of reference (i.e., a *stationary* observer looking from the platform's side), the forces acting on you are shown in Figure 5.33b. Using N.S.L., we can write:

$$\underline{\sum F_x}:$$

$$T = m(a_x)$$

$$= m(v^2/R)$$

$$\Rightarrow \quad v = (TR/m)^{1/2}.$$

f.b.d. looking from inertial frame of reference located on the side

N

T

+x

mg

(ignore frictional force into page)

FIGURE 5.33b

b.) From a non-inertial frame of reference (i.e., someone sitting next to you on the platform), the forces acting on you are shown in Figure 5.33c. Noting that as far as you are concerned, you aren't accelerating at all (you don't seem to be moving along the line of the tether), we can use N.S.L. to write:
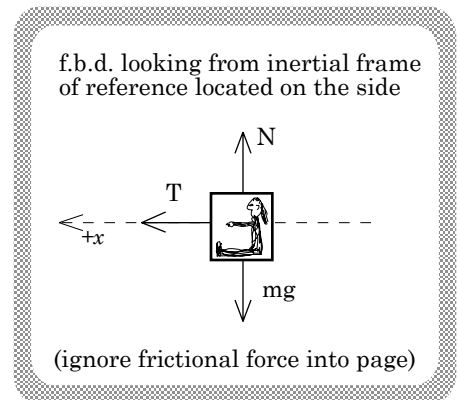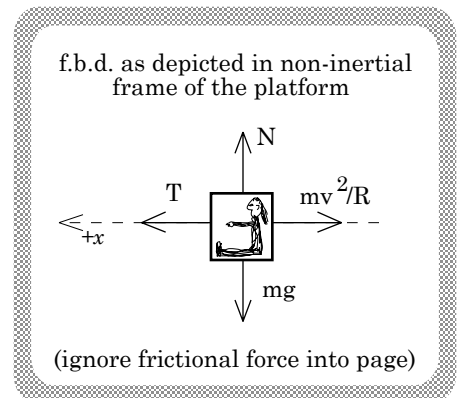
f.b.d. as depicted in non-inertial frame of the platform

N

T mv^2/R

+x

mg

(ignore frictional force into page)

FIGURE 5.33c

$\underline{\Sigma F_x}$:

$\quad$ T - m(v^2/R) = ma$_x$ = 0 $\qquad$ (as a$_x$ = 0 in this frame)

$\quad\Rightarrow\quad$ v = (TR/m)$^{1/2}$.

c.) The velocities determined in both frames are the same.

Note: The trick is to make the actual *ma* value, as observed in the inertial frame (this would be *mv^2/R* in this case), into a fictitious force in the non-inertial frame, then to look to see if there appears to be an apparent acceleration in the non-inertial frame. In the *car rounding the corner* example, the box appeared to be accelerating away from you, so in that case, there is an acceleration as viewed in the non-inertial frame of your car. In the above example, there doesn't appear to be acceleration in the non-inertial frame as you move around the circle, so *a = 0* in that case. In both cases, though, you took the real, inertial acceleration (for these cases, *v^2/R*), multiplied it by *m*, and treated it as though it were a force.

5.) One more *fictitious force* example: A mass on a spring is placed in an accelerating rocket (Figure 5.34a) out in space (i.e., no gravity). The rocket's acceleration is known to be *A*. Assuming you also know the spring constant, by how much is the spring elongated from its equilibrium position (the variable *y* in the sketch)? Analyze the situation from both the inertial frame of someone watching from outside the rocket, and then from the non-inertial frame of someone sitting inside the rocket with the spring.

accelerating rocket in space

equilibrium

y

m

FIGURE 5.34a

a.) From an inertial frame of reference (i.e., a stationary observer outside the rocket), the only force accelerating *m* in this gravity free situation is *ky* (Figure 5.34b). Using N.S.L., we can write:

$\underline{\Sigma F_y}$:

$\quad$ ky = m(a$_y$)

$\qquad$ = m(A)

$\quad\Rightarrow\quad$ y = (mA/k).

ky

f.b.d. on mass relative to inertial frame outside rocket
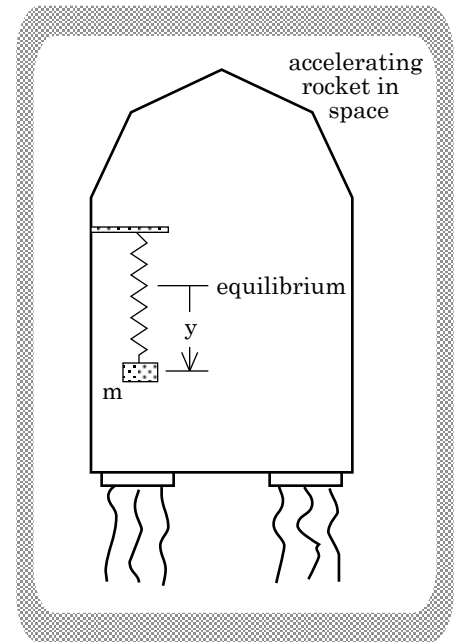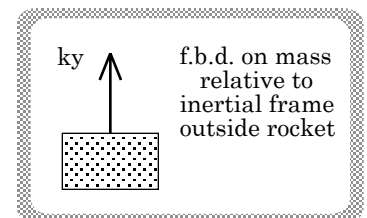
FIGURE 5.34b

b.) From a non-inertial frame of reference (i.e., inside the rocket), there are two apparent forces acting: ky pulling up and a fictitious force equal to mA pulling the spring down (Figure 5.34c). Noting that m is not accelerating relative to the rocket, N.S.L. yields:

FIGURE 5.34c

$$\underline{\Sigma F_y}:$$

$$ky - m(A) = 0 \qquad \text{(as } a = 0 \text{ as viewed from}$$

inside the frame)

$$\Rightarrow \quad y = (mA/k).$$

c.) Huzzah! The y values calculated in both cases are the same.

d.) Now re-read the first sentence of the Note just above *G-5*.
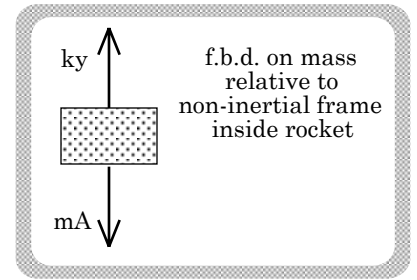
QUESTIONS

5.1) A car moves at a constant 30 m/s. Is it accelerating? If so, must there be an applied net force to maintain this motion? Explain.

5.2) Two objects of different mass will have different weights (that is, they will feel different gravitational forces), yet if air friction is ignored and you drop both from the same height, they will accelerate due to gravity at the same rate. How can this be?

ignoring air friction
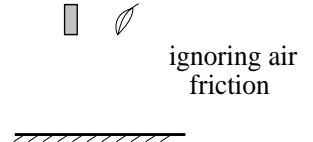
5.3) A solid copper ball and a hollow copper ball of the same radius are found in space. Both are weightless. Without cutting them open, how can you determine which is which?
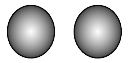
5.4) A body is accelerated by some net force. If the force is halved, how will the velocity-change Δv alter? If the mass is halved instead, how will the velocity-change Δv alter?

5.5) It is easier to keep a crate moving across a frictional floor than it is to get it going in the first place. Why? That is, aside from the relative motion, what is the fundamental difference between the two cases?

5.6) A heavy box attached to a parachute will reach the ground faster than a light box of equal size attached to an identical parachute. Why?

5.7) A truck is ten times more massive and moves with twice the speed of a small car. The two collide. During the collision, which will experience the greater force? Which will experience the greater acceleration?

2v

v

5.8) A man stands wedged between two identical crates on a frictionless sheet of ice. Is there any way he can make the acceleration of one of the blocks greater than that of the other?
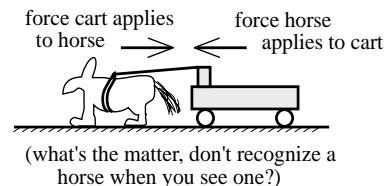
frictionless
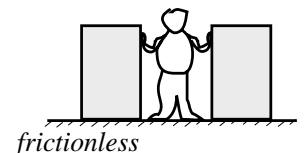
5.9) Drop a rock from the mast of a moving boat. Will it hit the deck a.) in front of the mast, b.) next to the mast, or c.) behind the mast? Justify your response in terms of the force(s) acting on the rock.

5.10) A kid steps off a footstool and begins to freefall under the influence of gravity. That is, the earth applies a gravitational force to the kid. For this situation, what is the "reaction force" alluded to in Newton's Third Law?

5.11) A force F_1 stops a car. In terms of F_1, how large must a new force be to stop the same car under the same circumstance but in half the distance? In half the time?

5.12) A horse pulls on a cart. According to Newton's Third Law, the cart must exert an equal and opposite reaction force back on the horse. That is, if the horse pulls the cart with 50 newtons of force, the cart must pull back on the horse with 50 newtons of force. As this so-called action/reaction pair always adds to zero, it appears as though we are suggesting that the cart will never accelerate. This obviously can't be the case, so what's the problem here?

force cart applies to horse → ← force horse applies to cart

(what's the matter, don't recognize a horse when you see one?)

5.13) Assuming friction is negligible, which will reach the bottom of an incline first, a large box or a small box? Explain.

5.14) An object falls from rest into a syrupy fluid. What does its *net force versus time* graph look like? Its *velocity versus time* graph? Its *position versus time* graph?

5.15) Assuming all masses are the same size and the pulleys are ideal (i.e., massless and frictionless), which of the three scales in the figure will register the greatest force?

5.16) A ball is dropped from rest a distance h units above a bathroom scale. When it hits, the scale measures a force of 50 newtons. The ball is then dropped from a distance $2h$ units above the scale. Will the scale read 100 newtons, less than 100 newtons, or more than 100 newtons? Explain.

scale

scale

scale

pulley

m

m m

m
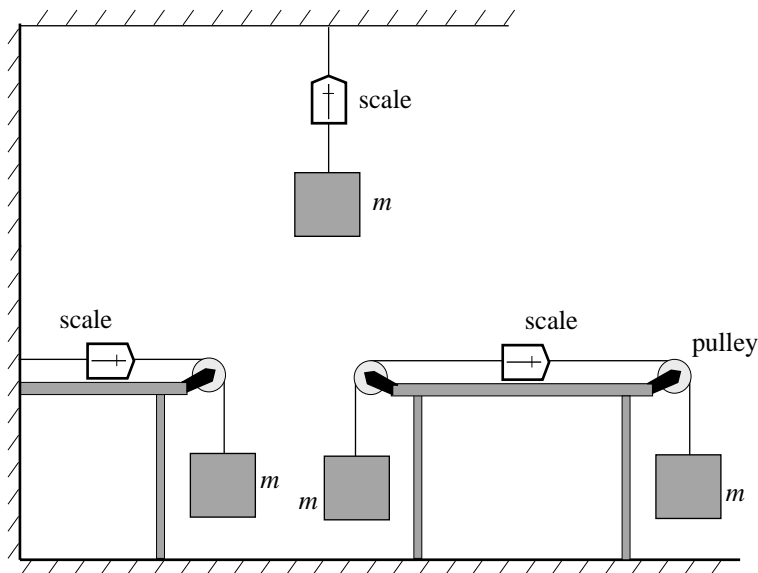
5.17) A massive object is placed on a frictionless table. It takes 2 newtons of force to accelerate it at .5 m/s^2. The object is taken into space where it is

weightless. The force required to accelerate the object at .5 m/s^2 will be (a) less than, (b) equal to, or (c) more than 2 newtons.

5.18) Six Flags Magic Mountain is an amusement park in Southern California that is known for giant roller coaster rides. One of the rides, Superman, consists of a cart that is accelerated along a horizontal stretch of track (via magnetic induction, no less) to somewhere around 100 mph in less than four seconds. The cart's path then curves into a vertical climb up an enormously high tower. At the top it comes to rest whereupon it proceeds to freefall several hundred feet back down the vertical section of the track and out the curve onto the horizontal section where it finally comes to rest. The drop from the top is billed as *pure freefall*. In theory, if you took this ride and released a dime at the top (i.e., just as you began to fall back down the track), the dime should sit motionless in front of you as both you and it gravitationally accelerated back toward Earth. For two reasons (one obvious and one not so obvious), DOING THIS WOULD BE A BAD IDEA--A REALLY, REALLY BAD IDEA. What is the obvious problem and, for the hotshots, what is the not so obvious problem?

5.19) A pendulum in Los Angeles (22° latitude) does not hang directly toward the center of the earth. Explain why not.

5.20) A block on a frictional incline plane compresses an ideal spring by a distance d. The spring is released firing the block up the incline. Old George maintains that the block will go up the incline and, upon returning, will re-compress the spring by some distance *less than d*. Why, in the real world, might he think that, and what additional, exotic thing might happen that could prove him wrong?

5.21) A mass m is attached to one end of a string. The other end of the string is attached to the ceiling of an elevator. The elevator proceeds from the first floor to the sixtieth floor. What might you expect the graph of the string tension to look like, relative to the force mg, as the motion proceeds? Being the big-hearted guy that I am, I'll give you a hint: the tension IS mg before the elevator begins to move. Justify each part of your graph.
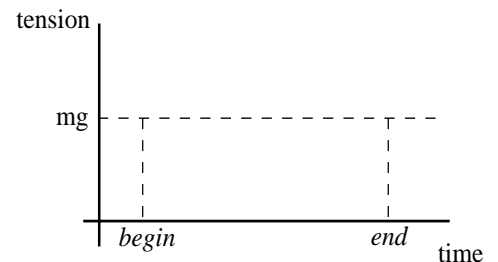
5.22) A block on a horizontal frictional surface is pulled by a rope oriented at some non-zero angle, relative to the horizontal. Is there an optimal angle at which the block's acceleration will be at a maximum and, if so, how would you go about theoretically determining that angle?

5.23) You want to set up the following device: Place a string over a pulley, then attach unequal masses to each end (this device is called *an Atwood Machine*--typically, the question asked for such a device is, "What will the acceleration of the system be if allowed to free-fall?"). Unfortunately, you are symmetrically challenged. Every time you thread the string over the pulley (i.e., before you get the masses attached), you put more string on one side of the pulley than on the other side. That means that when you release the string to pick up the masses, the string free-wheels over the pulley and ends up on the ground (really irritating).
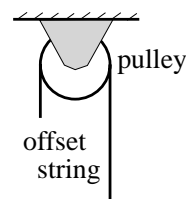
    You may be symmetrically challenged, but you aren't stupid. Realizing that you aren't going to be able to make the miserable thing work, you change the problem to *what is the <u>string's</u> freefall acceleration as it free-wheels over the pulley*? Without answering the question itself, answer the following: Will the acceleration be constant (i.e., could you use kinematics on this if you were clever?) and, if not, what parameters (i.e., height above the ground, initial velocity, what?) will determine what the string's acceleration is at a given instant?

5.24) A net force of 2 newtons is applied to a mass. The speed of the object doesn't change. How can this be?

5.25) The graph shown depicts an object's change of velocity with time. What might the unknown function be?
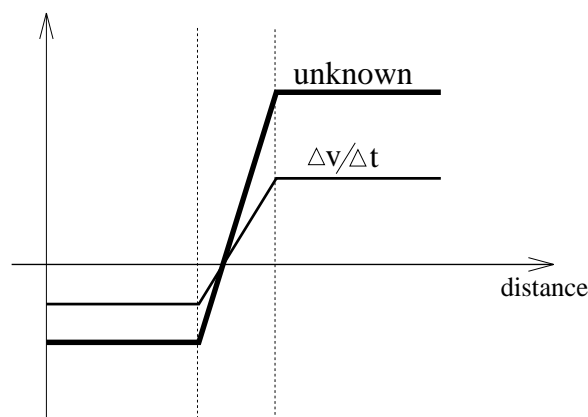
5.26) John was big, but he wasn't too bright. He needed to transport several 60 pound cubical microwave cookers (each was enameled plastic with no feet) across town in his car, but his beat up, rusty hulk of a vehicle was just a little too small. To accommodate the last two, he put them side by side on the roof. Once there, he urged the car forward only to find that they both broke loose and slid when he stopped. To remedy the problem, he put one on top of the other thereby doubling the normal force thereby doubling the frictional force. What problem is he likely to run into?

5.27) Three identical springs are attached at the ceiling. A bar of mass m is hooked to the group. If the new system's equilibrium position is d units below the springs' unstretched lengths, what must the spring constant be for each spring?
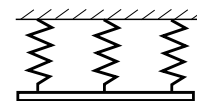
PROBLEMS

5.28) Draw a *free body diagram* for the forces acting on each of the bodies in each of the four independent sketches shown in Figure I. Label as completely as you can. Note that in *Part d, you are being asked to draw f.b.d.s on pulleys. This is for *practice in force recognition*, not because it is something you will ever need to do again*. Also, note that in that part, the pin supporting each pulley provides a force.
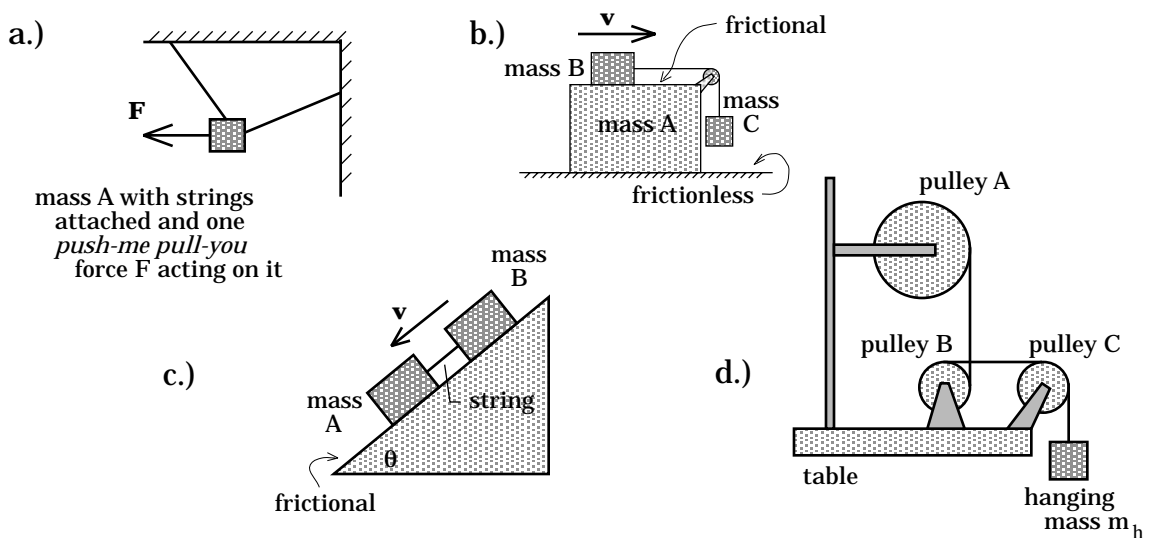
a.)

F

mass A with strings
attached and one
push-me pull-you
force F acting on it

b.) **v** frictional

mass B

mass A mass C

frictionless

c.)

mass
B

v

mass
A string

θ

frictional

d.)

pulley A

pulley B pulley C

table

hanging
mass m_h

FIGURE I

5.29) In each case below, state the "reaction force" that must exist according to Newton's Third Law.
        a.) The floor applies a force to you;
        b.) The string applies a force to the weight;
        c.) The car applies a force to the tree;
        d.) The earth applies a force to the moon.

Important Note: Getting numerical answers for the problems below is not particularly important. You have been given numbers to work with because numbers make students feel secure, but you should initially do each problem algebraically before sticking the numbers in (that is the way you will find the Solutions presented). WHAT IS IMPORTANT is that you can *follow the process* needed to approach and solve these problems, not the bottom line.

5.30) A sled whose mass is *m = 30 kg* slides horizontally with velocity magnitude *v = 2 m/s*. It runs into *sticky snow* and experiences a constant *frictional force* of f_k *= 12 nts*.

> **a.)** What is the sled's acceleration in the sticky snow? (Note that Figure II is for *Part b* only--there is no force *F* acting in *Part a*).
>
> **b.)** Your friend decides to help you by pulling the sled with a force *F = 60 newtons* at *θ = 40°* with the horizontal (Figure II). For this case, what is the acceleration of the sled? (Note that the vertical component of *F* will lighten the load, so to speak, lessening *N* and the *N-related* frictional force--you'll need to determine the coefficient of friction using the information available in *Part a*, then go from there).
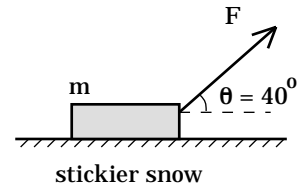

stickier snow

FIGURE II

5.31) An elevator whose mass is *m = 400 kilograms* is supported by a cable. A frictional force of *f = 80 newtons* is applied to the elevator as it moves. Determine the tension *T* in the cable when the elevator is:

> **a.)** Stationary;
>
> **b.)** Moving upward while accelerating at 2.8 m/s^2;
>
> **c.)** Moving upward with a constant velocity-magnitude of 5 m/s;
>
> **d.)** Accelerating downward at 2.8 m/s^2 with velocity downward;
>
> **e.)** Moving downward with a constant velocity-magnitude of 5 m/s.

5.32) You are standing on a bathroom scale in an elevator (yes, this is an odd place to find a bathroom scale). Your mass is *m = 60 kilograms*; the scale reads *W = 860 newtons*. What is the acceleration of the elevator?

5.33) You are sitting in a commercial jetliner. It is on the runway and ready to take off. Before it does, clever soul that you are, you hang a .5 meter long string from the ceiling just overhead and attach to its free end a weight whose mass is *m = .05 kg*. The jet begins its acceleration. As it does, the string and mass swing toward you until the string comes into equilibrium at a constant angle of *θ = 26°* with the vertical (see Figure III).
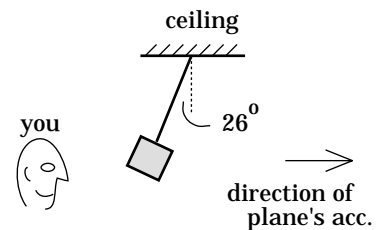

FIGURE III

> **a.)** What is the jet's acceleration?

b.) Once the jet gains altitude and proceeds with a constant velocity, what will the string and mass be doing (that is, will the 26° angle have changed)? Explain.

5.34) IMPORTANT PROBLEM: Two blocks of mass $m_1 = 2$ *kgs* and $m_2 = 7$ *kgs* are wedged up against one another and against a wall by a horizontal force F (see Figure IV to the right). Doing each section algebraically before putting in the numbers:

FIGURE IV

    a.) What is the *coefficient of static friction* between m_1 and m_2 (call this $\mu_{s,1}$) and between m_2 and *the wall* (call this $\mu_{s,2}$) if the <u>MINIMUM</u> FORCE F_{min} required to keep the blocks from breaking loose and sliding under the influence of gravity if $F_{min} = 25$ *newtons?*

    b.) The force F is decreased to 20 newtons. The blocks break loose and begin to fall. If the *coefficients of <u>kinetic</u> friction* between m_1 and m_2 AND between m_2 and *the wall* are $\mu_{k,1} = .15$ and $\mu_{k,2} = .9$ respectively, what are the accelerations of m_1 and m_2? Note that they will NOT be the same.
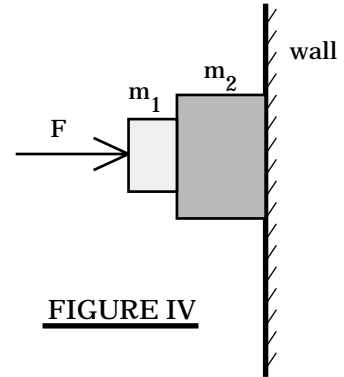
    c.) Why are the accelerations different?

5.35) IMPORTANT PROBLEM: Known masses $m_1 > m_2$ are attached by a string while sitting on an incline of known angle θ (see Figure V). If the coefficient of friction is a known μ_k and if the two are initially sliding UP the incline (no, you don't know *how* they managed it--somebody obviously gave them a push somewhere along the line--what matters is that they are moving UP the incline *when you see them*):

FIGURE V

    a.) Draw an f.b.d. for the forces acting on m_1 and m_2.

    b.) Derive an expression for the *acceleration* of the system (that is, the acceleration of either m_1 or m_2--both will be the same) as shown.

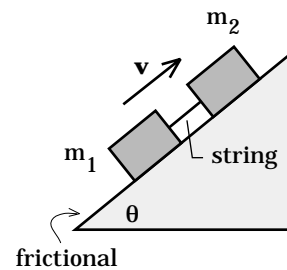   **c.)** Derive an expression for the *tension* in the string as shown.

5.36) IMPORTANT PROBLEM: A crate whose mass is m_1 is placed on an incline whose angle is θ. The *coefficient of friction* between the crate and the incline is a known μ_k. A wire attached to the mass proceeds over a frictionless, massless pulley and is attached to a hanging mass m_2. At the instant pictured in Figure VI, m_1 is moving *down* the incline. Additionally, at the instant shown the wire is at an angle ϕ with the line-of-the-incline. *For the moment shown* in the figure, derive general algebraic expressions from which you could determine the crate's acceleration. <u>DO NOT SOLVE THESE EQUATIONS</u>, just generate them!
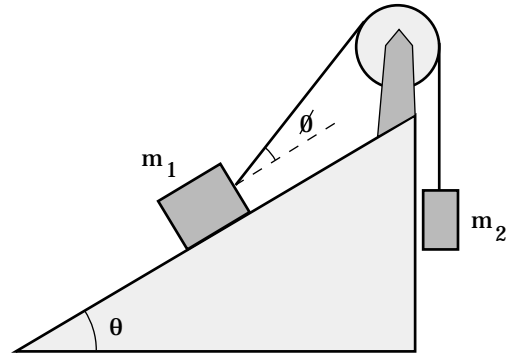
Hint: This is not as hard as it looks. <u>Do not look ahead</u>! Content your-self with *following the steps* you have been given in an orderly fashion. This problem will fall out nicely if you do it in pieces. (See additional note below!)

NOTE: This problem is designed to make you think about NSL, but it has one unfortunate characteristic that is not typical of all NSL problems. The acceleration of m_2 is in the same direction and has the same magnitude as the acceleration of the *string* attached to it (this is called a_{string} in the sketch). Unfortunately, the string is attached to m_1 *at an angle*, so m_1's acceleration and the
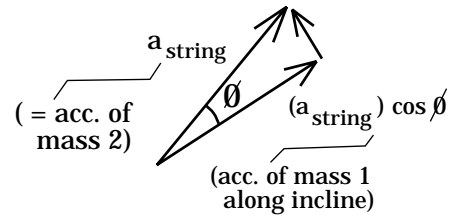
string's acceleration aren't the same. If we call m_1's acceleration a_1 and m_2's acceleration a_2, the relationship between the two parameter's is $a_1 = a_2 \cos \phi$. I'm pointing this out because it is very much a side issue. I don't want you spending a lot of time stewing over it.

FIGURE VI

FIGURE VI, supplemental

5.37) A mass m_1 sitting on a frictionless table is attached to a string. The free end of the string is threaded through a hole in the table. Once through, it is attached to a hanging mass m_2 (see Figure VII). If m_1 is then given a shove that moves it into circular
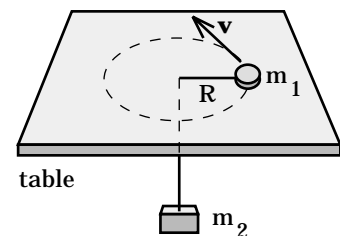
FIGURE VII

motion, what must its velocity be if its radius is to be R?

5.38) The Loop is a favorite Magic Mountain roller coaster ride that has within it a complete, vertical, 360° loop. The carts that travel through the loop are attached to the track; let's assume one comes loose. What is the *minimum velocity* magnitude required for such a cart to make it through the top of the loop without coming off the track?

   Note: At minimum velocity, the cart will just barely free fall through the top of the arc. In that case, the *normal force* applied to the cart by the track will be zero.

5.39) A particular brand of string can withstand a force of $T = 50$ *newtons* of tension before breaking. A rock of mass $m = .2$ *kg* is attached to a string of length $L = 1.2$ *meters*. A kid then takes the *string and mass* and whirls them around her head (see Figure VIII).
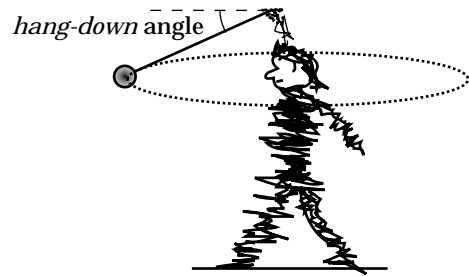

hang-down angle

FIGURE VIII

   a.) Assuming the radius of motion is, to a good approximation, equal to the string's length (that is, assume the mass is moving very fast and the *hang-down* angle is zero), what is the *maximum speed* at which the rock can rotate before the string breaks?

   b.) Taking the hang-down into consideration (i.e., assume the string's length is no longer equal to the radius of the rock's motion), what is the angle at which the string breaks?

   c.) Use the angle determined in *Part b* to determine the velocity of the mass in that situation.

   d.) The rock is observed to be hanging at a 30° angle. How fast is it moving?

5.40) Until now, we have dealt only with gravitational forces between the earth and small objects close to the earth's surface. Under this circumstance, we have taken the *acceleration of gravity* to be nearly constant and equal to $g = 9.8 \ m/s^2$. Newton realized that gravity must exist between *any* two objects no matter how far apart they were. To accommodate the most general situation, he derived a general *gravitational force* expression the magnitude of which is:

$$F_g = G \, m_1 m_2 / r^2.$$

In this equation, G is called the *Universal Gravitational Constant* and is equal to 6.67×10^{-11} m^3/kg·s^2; m_1 and m_2 are the masses generating the force field, and r is the distance between the *center of mass* of the two objects.

    a.) The earth has a *mass* $m_e = 5.98 \times 10^{24}$ *kg* and a *radius* $r_e = 6.37 \times 10^6$ *meters*. Assuming *your* mass is 70 kg and you are standing on the earth's surface, what is the force of attraction between you and the earth according to Newton's general expression?

    After finishing this calculation, determine the gravitational force between you and the earth, given the assumption that your weight equals *mg*. How do the two numbers compare?

    b.) The *acceleration of gravity* on the earth's surface at the poles is 9.83 m/s^2. What is the *acceleration of gravity* at the equator? (Think *centripetal acceleration*.)

    c.) The moon has a mass $m_m = 7.35 \times 10^{22}$ *kg* and an orbital distance from the earth's center of 3.84×10^8 meters. What must its orbital speed be?